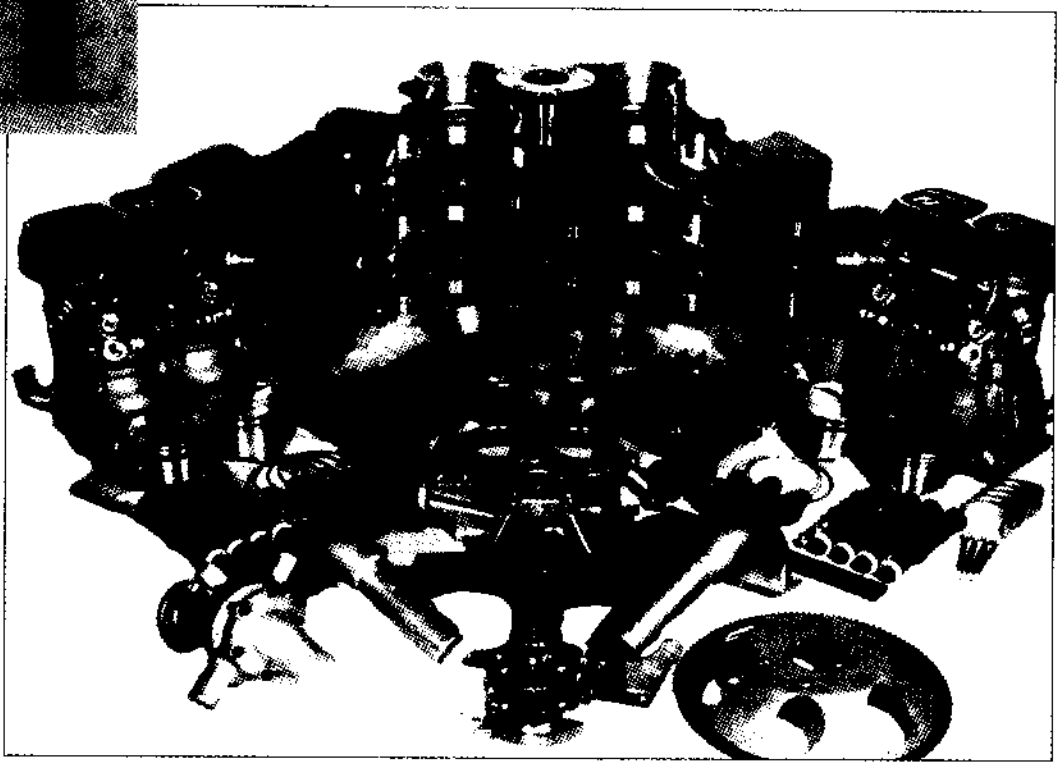
1



MECHANISMS

1.1 INTRODUCTION

A mechanism is a set of machine elements or components or parts arranged in a specific order to produce a specified motion. The machine elements or components are considered rigid or resistant bodies which do not deform under the action of forces. Resistant bodies are those bodies that do not suffer appreciable distortion or change in physical form due to forces acting on them, for example springs, belts, fluids etc. Elastic bodies are also resistant bodies. They are capable of transmitting the required forces with negligible deformation. Rigid bodies are those bodies that do not deform under the action of forces. All resistant bodies are considered rigid bodies for the purpose of transmitting motion. In this chapter, we shall study the different ways of connecting rigid (resistant) bodies to obtain various types of mechanisms.

Kinematics is a subject that deals with the study of relative motion of parts constituting a machine, neglecting forces producing the motion. A structure is an assemblage of a number of resistant bodies meant to take up loads or subjected to forces having straining actions, but having no relative motion between its members. A frame is a structure which supports moving parts of a machine.

1.2 ELEMENTS OR LINKS

A link (or element or kinematic link) is a resistant body (or assembly of resistant bodies) that constitutes the part (or parts) of the machine connecting other parts which have motion relative to it. A slider crank mechanism of an internal combustion engine, shown in Fig.1.1, consists of four links, i.e. frame 1, crank 2, connecting rod 3 and slider 4.

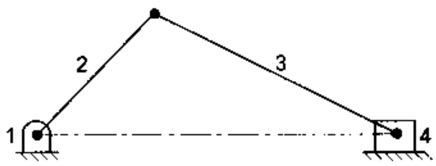


Fig.1.1 Slider–crank chain

1.2.1 Classification of Links

Links can be classified as binary, ternary, quaternary etc. depending upon the ends on which revolute or turning pairs can be placed, as shown in Fig.1.2. A binary link has two vertices, a ternary has three vertices, ·ι quaternary link has four vertices and so on.

(a) Binary link (b) Ternary link (c) Quaternary link

Fig.1.2 Types of links

Tl 'e are four types of links: rigid, flexible, fluid and floating links.

Ri d link A rigid link is one which does not undergo any deformation while transmitting motion. Links in :neral are elastic in nature. They are considered rigid if they do not undergo appreciable deformation wl ɜ transmitting motion, for example connecting rod, crank, tappet rod etc.

Fl ible link A flexible link is one which while transmitting motion is partly deformed in a manner not to af :t the transmission of motion, for example belts, ropes, chains, springs etc.

F' id link A fluid link is one which is deformed by having a fluid in a closed vessel and the motion is ' ansmitted through the fluid by pressure, as in the case of a hydraulic press, hydraulic jack and fluid brake.

Floating link This is a link which is not connected to the frame.

1.3 KINEMATIC PAIRS

Two links of a machine, when in contact with each other, are said to form a pair. A kinematic pair consists of two links which have relative motion between them. In Fig.1.1, links 1 and 2, 2 and 3, 3 and 4 and 4 and 1 constitute kinematic pairs.

Kinematic pairs may be classified according to the following considerations:

- Type of relative motion
- Type of contact
- Type of mechanical constraint

Kinematic pairs according to the relative motion

Sliding pair This consists of two elements connected in such a manner that one is constrained to have sliding motion relative to another; for example, a rectangular bar in a rectangular hole, the piston and cylinder of an engine, the cross-head and guides of a steam engine, the ram and its guides in a shaper, the tailstock on the lathe bed etc.

Turning (revolute) pair This consists of two elements connected in such a manner that one is constrained to turn or revolve about the fixed axis of another element; for example, a shaft with a collar at both ends revolving in a circular hole, a crankshaft turning in a bearing, cycle wheels revolving over their axles etc.

Rolling pairs When two elements are so connected that one is constrained to roll on another element which is fixed, they form a rolling pair. Ball and roller bearings, a wheel rolling on a flat surface etc. are examples of rolling pairs.

Screw (or helical) pair When one element turns about the other element by means of threads, they form a screw pair. The motion in this case is a combination of sliding and turning. The lead screw of a lathe with a nut, a bolt with a nut, a screw with the nut of a jack etc. are some examples of screw pairs.

Spherical pair When one element in the form of a sphere turns about the other element which is fixed, they form a spherical pair. The ball and socket joint, a pen stand, the mirror attachment of vehicles etc. are some examples of spherical pairs.

Kinematic pairs according to the type of contact

Lower pair When the two elements of a pair have surface (or area) contact while in motion and the relative motion is purely turning or sliding, the pair is called a lower pair. All sliding pairs, turning pairs and screw pairs form lower pairs; for example, a nut turning on a screw, a shaft rotating in a bearing, an universal joint, all pairs of a slider crank mechanism, a pantograph etc.

Higher pair When the two elements have point or line contact while in motion and the relative motion is a combination of sliding and turning, then the pair is known as a higher pair. Belt, rope and chain drives, gears, the cam and follower, ball and roller bearings, a wheel rolling on a surface etc. all form higher pairs.

Kinematic pairs according to the type of mechanical constraint

Closed pair When the two elements of a pair are held together mechanically in such a manner that only the required type of relative motion occurs, they are called a closed pair. All lower pairs and some higher pairs (for example, the enclosed cam and follower) are closed pairs.

Unclosed pair When the two elements of a pair are not held mechanically and are held in contact by the action of external forces, the pair is called an unclosed pair, for example, the cam and spring loaded follower pair.

1.4 CONSTRAINED MOTION

The three types of constrained motion are:

Completely constrained motion When the motion between a pair takes place in a definite direction irrespective of the direction of force applied, then it is said to be a completely constrained motion. For example, a square bar in a square hole, a shaft with collars at each end in a circular hole, a piston in the cylinder of an internal combustion engine, all have completely constrained motion.

Partially (or successfully) constrained motion When constrained motion between a pair is not completed by itself but by some other means, it is called successfully constrained motion. For example, the motion of a shaft in a footstep bearing becomes successfully constrained motion when compressive load is applied to the shaft.

Incompletely constrained motion When motion between a pair can take place in more than one direction, it is called incompletely constrained motion, for example, a circular shaft in a circular hole.

1.5 KINEMATIC CHAIN

A kinematic chain may be defined as an assembly of links in which the relative motion of the links is possible and the motion of each link relative to the others is definite. The last link of the kinematic chain is attached to the first link. The four-bar mechanism and the slider–crank mechanism are some examples of a kinematic chain.

The following relationship holds for a kinematic chain having lower pairs only:

$$N = 2P - 4 \tag{1.1a}$$

$$J = \frac{3N}{2} - 2 \tag{1.1b}$$

where N = number of links

P = number of pairs

J = number of joints.

If LHS > RHS, then the chain is locked

LHS = RHS, then the chain is constrained

LHS < RHS, then the chain is unconstrained

For a kinematic chain having higher pairs, each higher pair is taken as equivalent to two lower pairs and an additional link.

1.6 TYPES OF MECHANISMS

When one of the links of a kinematic chain is fixed, the chain is called a mechanism. Mechanisms are of the following types:

Simple mechanism This is a mechanism which has four links.

Compound mechanism This is a mechanism which has more than four links.

Complex mechanism This is formed by the inclusion of ternary or higher order floating link to a simple mechanism.

Planar mechanism This is formed when all the links of the mechanism lie in the same plane.

Spatial mechanism This is formed when the links of the mechanism lie in different planes.

Equivalent mechanism Turning pairs of plane mechanisms may be replaced by other types of pairs such as sliding pairs or cam pairs. The new mechanism thus obtained having the same number of degrees of freedom as the original mechanism is called the *equivalent mechanism*. For instance,

- A turning pair can be replaced by a sliding pair.
- A spring can be replaced by two binary links.
- A cam pair can be replaced by one binary link with two turning pairs at each end.

1.7 MECHANISM AND MACHINES

A machine is a device that transforms energy available in one form to another to do certain type of desired useful work. The parts of the machine move relative to one another. Its links may transmit both power and motion. On the other hand, a mechanism is a combination of rigid or restraining bodies which are so shaped and connected that they move upon each other with definite relative motion. A mechanism is obtained when one of the links of the kinematic chain is fixed. A machine is a combination of two or more mechanisms arranged in such a way so as to obtain the required motion and transfer the energy to some desired point by the application of energy at some other convenient point. A machine is not able to move itself and must get the motive power from some source.

Some examples of mechanisms are the slider-crank, the typewriter, clocks, watches, spring toys etc. The steam engine, internal combustion engine, lathe, milling machine, drilling machine etc. are some examples of machines.

Machines may be classified as:

Simple machine In a simple machine, there is one point of application for the effort and one point for the load to be lifted. Some examples of simple machines are the lever, screw jack, inclined plane, bicycle etc.

Compound machine In a compound machine, there are more than one point of application for the effort and the load. A compound machine may be thought of as a combination of many simple machines. Some examples of compound machines are lathe machine, grinding machine, milling machine, printing machine etc.

1.8 DEGREES OF FREEDOM

An unconstrained rigid body moving in space can have three translations and three rotational motions (that is six motions) about three mutually perpendicular axes. The number of degrees of freedom of a kinematic pair is defined as the number of independent relative motions, both translational and rotational that a kinematic pair can have.

$$\text{Degrees of freedom} = 6 - \text{number of restraints} \tag{1.2}$$

The degrees of freedom of some of the systems are as follows:

- A rigid body has six degrees of freedom.
- A rectangular bar sliding in a rectangular hole has one degree of freedom as the motion can be expressed by the linear displacement only.
- The position of the crank of a slider–crank mechanism can be expressed by the angle turned through and thus has one degree of freedom.
- A circular shaft rotating in a hole and also translating parallel to its axis has two degrees of freedom, that is, angle turned through and displacement.
- A ball and a socket joint has three degrees of freedom.

1.8.1 Degrees of Freedom of Planar Mechanisms

Mobility of a mechanism The mobility of a mechanism is defined as the number of degrees of freedom it possesses. An equivalent definition of mobility is the minimum number of independent parameters required to specify the location of every link within a mechanism.

Kutzbach criterion The Kutzbach criterion for determining the number of degrees of freedom of a planar mechanism is,

$$F = 3(N-1) - 2P_1 - P_2 \tag{1.3}$$

where F = number of degrees of freedom

N = total number of links in a mechanism out of which one is a fixed link.

$N - 1$ = number of movable links

P_1 = number of simple joints or lower pairs having one degree of freedom

P_2 = number of higher pairs having two degrees of freedom

When two links are joined by a hinge, two degrees of freedom are lost. Hence, for each joint two degrees of freedom are lost. Therefore, for P_1 number of joints the number of degrees of freedom lost are $2P_1$. When a kinematic chain is made up of different types of links, then the number of lower pairs P_l is computed as follows:

$$P_l = \frac{1}{2}\left[2N_2 + 3N_3 + 4N_4 + \cdots\right] \tag{1.4}$$

where N_2 = number of binary links

N_3 = number of ternary links, and so on.

For linkages with lower pairs only, $P_2 = 0$, and

$$F = 3(N-1) - 2P_l \tag{1.5}$$

A joint connecting k links at a single joint must be counted as $(k-1)$ joints. Only four types of joints are commonly found in planar mechanisms. These are the revolute, the prismatic, the rolling contact joints (each having one degree of freedom), and the cam or gear joint (each having two degrees of freedom). These joints are depicted in Fig.1.3. The following definitions apply to the actual degrees of freedom of a device:

$F \geq 1$: the device is a mechanism with F degrees of freedom.

$F = 0$: the device is a statically determinate structure.

$F < -1$: the device is a statically indeterminate structure.

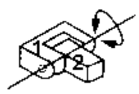
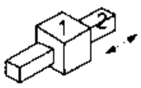
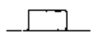
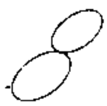
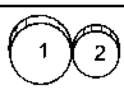
Joint type (Symbol)	Physical form	Schematic representation	Degrees of freedom
Revolute (R)			1 (Pure rotation)
Prismatic (P)			1 (Pure sliding)
Cam or gear			2 (Rolling and sliding)
Rolling contact			1 (Rolling without sliding)

Fig.1.3 Common types of joints found in planar mechanisms

The degrees of freedom of some of the planar mechanisms have been listed in Table 1.1.

Gruebler criterion For a constrained motion, $F = 1$, so that

$$1 = 3(N-1) - 2P_l - P_2$$

or

$$2P_l + P_2 - 3N + 4 = 0 \tag{1.6}$$

Equation (1.6) represents the Gruebler criterion.

If $P_2 = 0$, then

$$P_1 = \frac{3N}{2} - 2 \tag{1.7}$$

Therefore, a planar mechanism with $F = 1$ and having only lower pairs, cannot have odd number of links.

Table 1.1 Degrees of freedom of planar mechanisms

Mechanism	N	P_1	P_2	$F = 3(N-1) - 2P_1 - P_2$
1. Three-bar	3	3	0	0
2. Four-bar	4	4	0	1
3. Five-bar	5	5	0	2
4. Five-bar	5	6	0	0
5. Six-bar	6	8	0	−1
6. Four-bar	4	5	0	−1
7. Three-bar	3	2	1	1
8. Four-bar	4	3	1	2
9. Five-bar	5	$\frac{11}{2}$	0	1
10. Six-bar	6	7	0	1

1.9 FOUR-BAR CHAIN

A four-bar chain has been shown in Fig. 1.4. It consists of four binary links. Link *AD* is fixed (called frame), *AB* is the crank (or driver link), *BC* is the coupler (or connecting rod) and *CD* the lever (or rocker or follower link). The input angle is θ and ϕ the angle of transmission. The coupler *BC* may be a ternary link. The number of degrees of freedom of the four-bar chain is one.

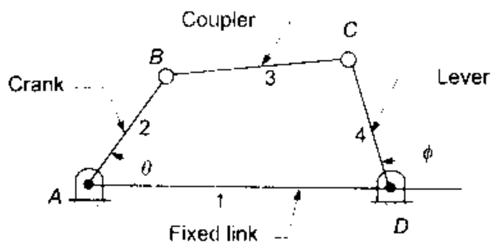


Fig.1.4 Four-bar chain

A link that makes complete revolutions is the crank, the link opposite the fixed link is the coupler, and the fourth link is called a lever or rocker, if it oscillates, or another crank, if it rotates.

The four-bar mechanism with all its pairs as turning pairs is called the quadric cycle chain. When one of these turning pairs is replaced by a slider pair, the chain becomes single slider chain. When two turning pairs are replaced by slider pairs, it is called a double slider chain or a crossed double slider chain, depending on whether the two slider pairs are adjacent or crossed.

1.10 GRASHOF'S LAW

This law states that for a four-bar mechanism the sum of the lengths of the largest and the shortest links should be less than the sum of the lengths of the other links, that is,

$$(l + s) < (a + b) \tag{1.8}$$

where l, s = lengths of the longest and the shortest links, respectively

a, b = lengths of the other links.

Consider the four-bar chain shown in Fig.1.5. Let the length of fixed link $O_2O_4 = l_1$, crank $O_2A = l_2$, coupler $AB = l_3$, and lever $BO_4 = l_4$. The following types of mechanisms are obtained by adjusting the lengths of various links:

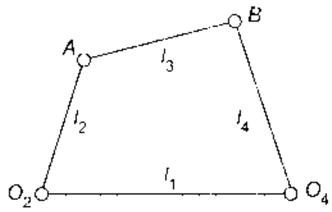


Fig.1.5 Four-bar chain

1.10.1 Crank–lever Mechanism

When for every complete revolution of link 2, link 4 makes a complete oscillation, the mechanism is called a crank–lever mechanism, as shown in Fig.1.6. Here

$$(l_2 + l_3) < (l_1 + l_4)$$

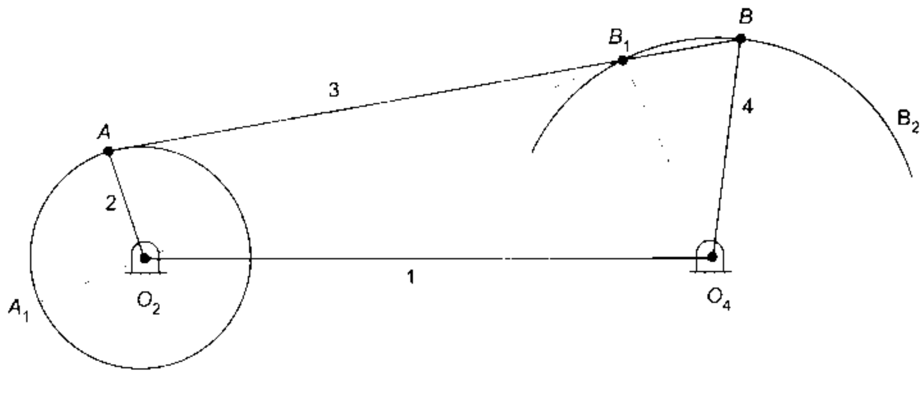


Fig.1.6 Crank–lever mechanism

1.10.2 Double Lever Mechanism

In this case, links 2 and 4 both can only oscillate.

Here
$$(l_3 + l_4) < (l_1 + l_2)$$
$$(l_2 + l_3) < (l_1 + l_4)$$

The double lever mechanism is shown in Fig.1.7.

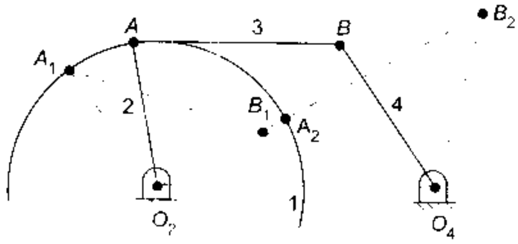


Fig.1.7 Double lever mechanism

1.10.3 Double Crank Mechanism

In this case, links 2 and 4 make complete revolutions. This mechanism is of two types.

Parallel–crank mechanism Here $l_1 = l_3$ and $l_2 = l_4$. The mechanism is shown in Fig.1.8.

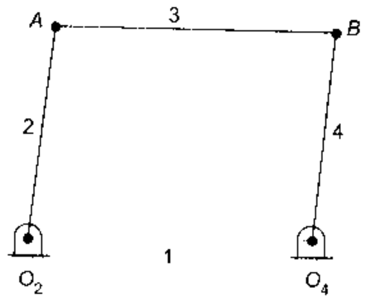


Fig.1.8 Parallel–crank mechanism

Drag link mechanism Here $l_3 > l_1$ and $l_4 > l_2$.

Also $\quad l_3 > (l_1 + l_4 - l_2)$

and $\quad l_3 < (l_2 + l_4 - l_1)$

The mechanism is shown in Fig.1.9.

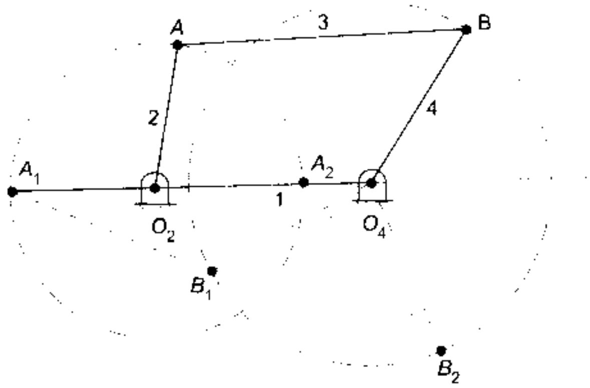


Fig.1.9 Drag link mechanism

1.11 INVERSION OF MECHANISMS

A kinematic chain becomes a mechanism when one of its links is fixed. Therefore, the number of mechanisms that can be obtained is as many as the number of links in the kinematic chain. This method of obtaining different mechanisms by fixing different links of a kinematic chain is called inversion of the mechanism. The relative motion between various links is not altered as a result of inversion, but their absolute motion with respect to the fixed link may alter drastically.

1.11.1 Inversions of a Four-bar Chain

Some of the important inversions of a four-bar chain are the following: beam engine, coupled wheel of locomotive, Watt's indicator mechanism and the slider–crank chain.

Beam engine The beam engine mechanism is shown in Fig.1.10. It consists of four links. When the crank AB rotates about the fixed centre *A*, the lever oscillates about a fixed centre *D*. The end *E* of the lever *CDE* is connected to a piston rod which moves the piston up and down in the cylinder. This is also called crank and lever mechanism.

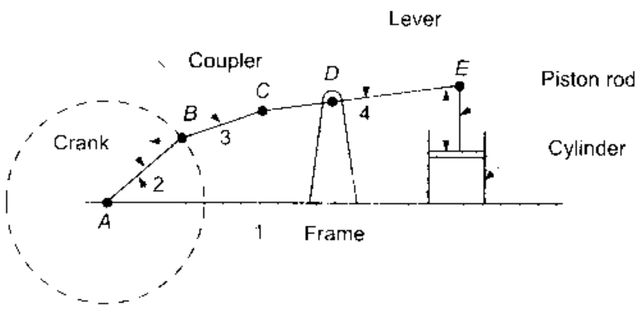


Fig.1.10 Beam engine

Coupled wheel of a locomotive In this mechanism, as shown in Fig.1.11, the links *AB* and *CD* are of equal lengths and act as cranks. These cranks are connected to the respective wheels. The link BC acts as the connecting rod. The link *AD* is fixed to maintain constant distance between the wheels. This mechanism is used to transmit rotary motion from one wheel to the other. This is also called the double crank mechanism.

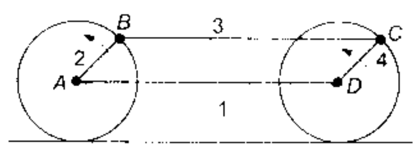


Fig.1.11 Coupled wheel of a locomotive

Watt's indicator mechanism This mechanism is shown in Fig.1.12. It consists of four links: a fixed link at *A*, link *AC*, link *CE* and link *BFD*. Links *CE* and *BFD* act as levers. The displacement of link *BFD* is directly proportional to the pressure in the indicator cylinder. The point *E* on link *CE* traces out an approximate straight line. It is also called double lever mechanism.

Single-slider–crank chain The single-slider crank chain shown in Fig.1.13 consists of three turning pairs and one sliding pair. Link 1 corresponds to the frame of the mechanism, which is fixed. Link 2 is the crank and link 3 the connecting rod. Link 4 is the slider. It is used to convert rotary motion into reciprocating motion and vice-versa.

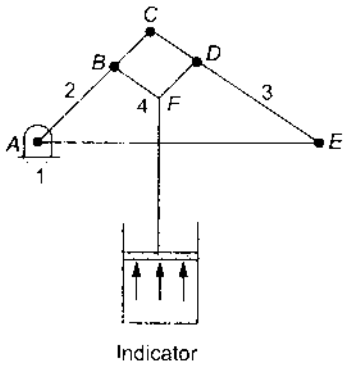


Fig.1.12 Watt's indicator mechanism

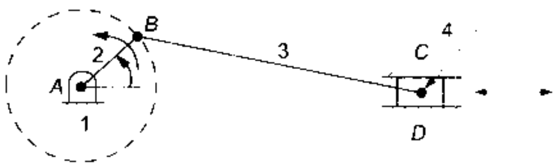


Fig.1.13 Single-slider–crank mechanism

1.11.2 Inversions of a Single-slider–crank Chain

The inversions of a single-slider–crank chain are pendulum pump, oscillating cylinder engine, rotary internal combustion engine, crank and slotted lever quick-return motion mechanism and Whitworth quick-return motion mechanism.

Pendulum pump This inversion mechanism is obtained by fixing link 4, that is, the sliding pair, as shown in Fig.1.14. When link 2 (that is, the crank) rotates, link 3 (that is, the connecting rod) oscillates about a pin pivoted to fixed link 4 at *C* and the piston attached to the piston rod (link 1) reciprocates in the cylinder.

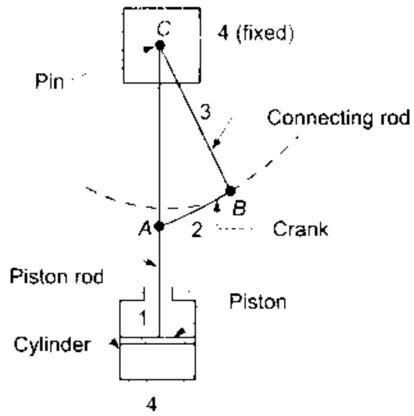


Fig.1.14 Pendulum pump

Oscillating cylinder engine In this mechanism, as shown in Fig.1.15, link 3 is fixed. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A.

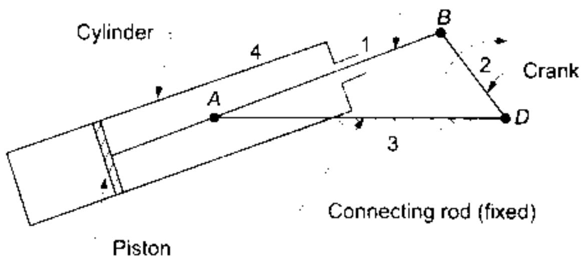


Fig.1.15 Oscillating cylinder engine

Rotary internal combustion engine (Gnome engine) It consists of several cylinders in one plane and all revolve about fixed centre O, as shown in Fig.1.16. The crank (link 2) is fixed. When the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinder forming link 1.

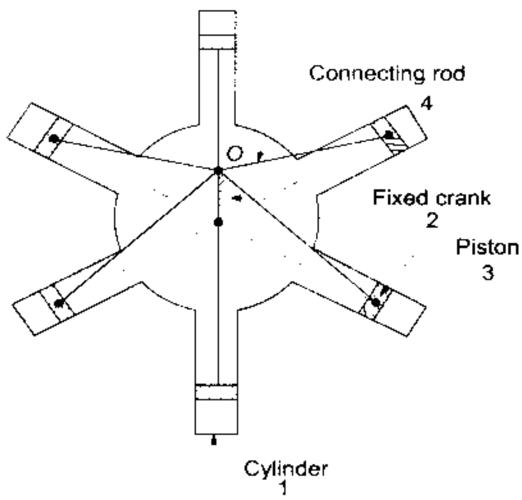


Fig.1.16 Rotary internal combustion engine

Crank and slotted lever quick-return motion mechanism In this mechanism, as shown in Fig.1.17, link AC (link 3) corresponding to the connecting rod is fixed. The driving crank CB (link 2) revolves about centre C. A slider (link 1) attached to the crank pin at B slides along the slotted lever AP (link 4) and make the slotted lever oscillate about the pivoted point A. A short link PQ transmits the motion from AP to the arm which reciprocates with the tool along the line of stroke. The line of stroke is perpendicular to AC produced. This mechanism is mostly used in shaping machines, slotting machine and rotary internal combustion engines.

$$\text{Time of cutting stroke / Time of return stroke} = \frac{\alpha}{\beta} \tag{1.9}$$

$$\text{Length of stroke} = 2AP\left(\frac{CB}{AC}\right) \tag{1.10}$$

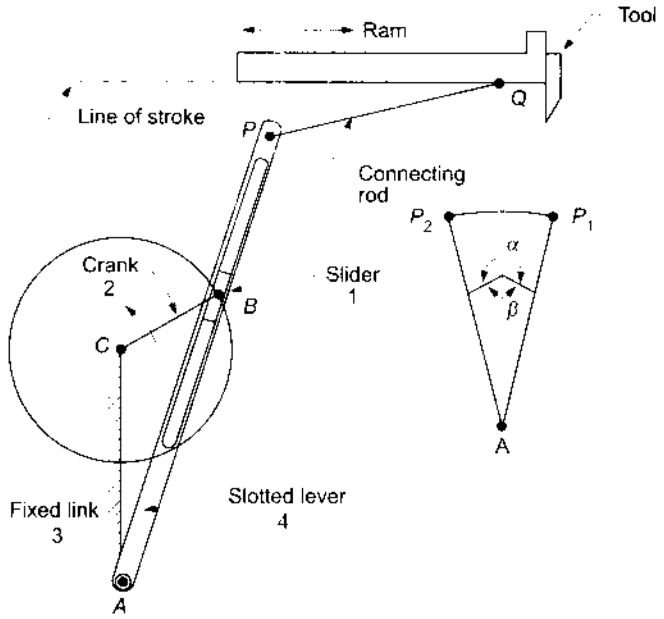


Fig.1.17 Crank and slotted lever quick-return motion mechanism

Whitworth quick return motion mechanism In this mechanism, as shown in Fig.1.18, link *CD* (link 2) is fixed. The driving crank *CA* (link 3) rotates about *C*. The slider (link 4) attached to the crank pin at *A* slides along the slotted lever *PA* (link 1) which oscillates about pivot *D*. The connecting rod *PQ* carries the ram at *Q* with cutting tool. The ram reciprocates along the line of stroke. It is used in shaping and slotting machines.

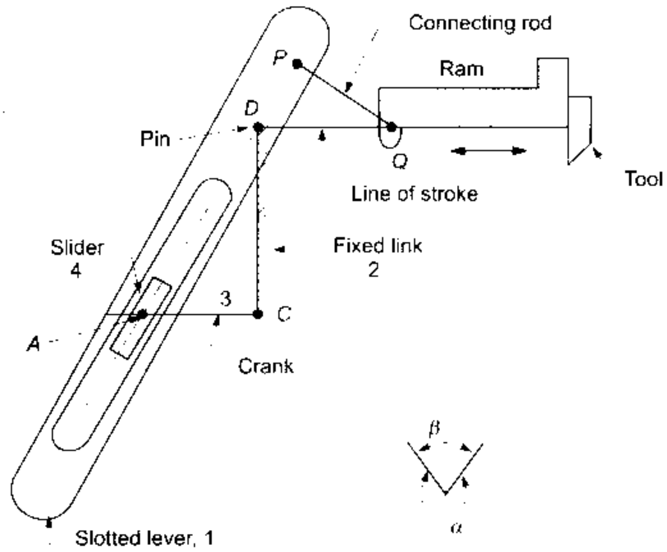


Fig.1.18 Whitworth quick-return motion mechanism

$$\text{Time of cutting stroke / Time of return stroke} = \frac{\alpha}{\beta} \qquad (1.11)$$

$$\text{Length of stroke} = 2PD \qquad (1.12)$$

Toggle mechanism This mechanism has many applications where it is necessary to overcome a large resistance with a small driving force. Fig.1.19 shows the toggle mechanism; links 4 and 5 are of equal length. As the angles α decrease and links 4 and 5 approach being collinear, the force F required to overcome a given resistance P decreases as:

$$F = 2P \tan \alpha \tag{1.13}$$

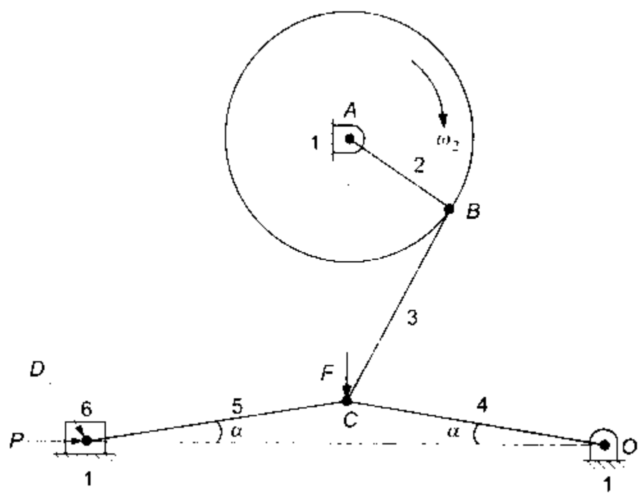


Fig.1.19 Toggle mechanism

If α approaches zero, for a given F, P approaches infinity. A stone crusher utilizes this mechanism to overcome a large resistance with a small force. It can be used in numerous toggle clamping devices, for holding work pieces.

The summary of single slider–crank chain and its inversions is given in Table 1.2.

Table 1.2 Summary of single slider–crank chain and its inversions

Mechanism	Links			
	Fixed	Rotates	Oscillates	Reciprocates
Single slider–crank chain link	1	2	3	4
Inversions:				
Pendulum pump	4	2	3	1
Oscillating cylinder engine	3	2	4	1
Crank and slotted lever	3	2	4	1
Whitworth mechanism	2	3	1	4
Gnome engine	2	3	1	4

1.12 DOUBLE SLIDER–CRANK CHAIN

A kinematic chain consisting of two turning pairs and two sliding pairs is called double slider–crank chain, as shown in Fig.1.20. Links 3 and 4 reciprocate, link 2 rotates and link 1 is fixed. The two pairs of the same kind are adjacent.

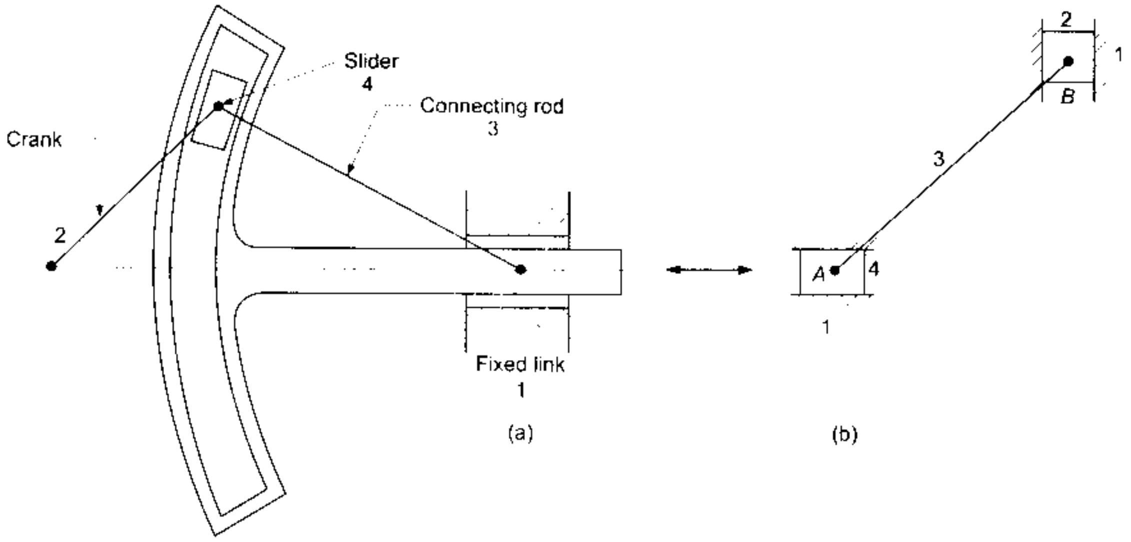


Fig.1.20 Double slider–crank chain

1.12.1 Inversions of Double Slider–crank Chain

The inversions of double slider–crank chain are the donkey pump, Oldham's coupling, the elliptical trammel and the Scotch yoke.

Donkey pump Figure 1.21 shows a donkey pump in which link 2 (crank) rotates about point *A*. One end of the crank is connected to the piston through the piston rod, which reciprocates vertically in the pump cylinder. This cylinder together with the body of the pump represent the fixed link 1. The other end of the crank is connected to the slider (link 1), which reciprocates horizontally in the cylinders.

Oldham's coupling Oldham's coupling shown in Fig.1.22, is used to connect two parallel shafts, the distance between whose axes is small and variable. The shafts connected by the coupling rotate at the same speed. The shafts have flanges at the ends, in which slots are cut. These form links 1 and 3. An intermediate piece circular in shape and having tongues at right angles on opposite sides, is fitted between the flanges of the two shafts in such a way that the tongues of the intermediate piece get fitted in the slots of the flanges. The intermediate piece forms link 4 which slides or reciprocates in links 1 and 3. Link 2 is fixed.

Maximum sliding speed of each tongue along its slot

= Distance between the axes of the shafts × Angular velocity of each shaft (1.14)

Elliptical trammel This is a device to draw ellipses. Fig.1.23 shows an elliptical trammel in which two grooves are cut at right angles in a plate which is fixed. The plate forms the fixed link 4. Two sliding blocks are fitted into the grooves. The slides form two sliding links 1 and 3. The link joining slides form the link 2. Any point on link 2 or on its extension traces out an ellipse on the fixed plate, when relative motion occurs.

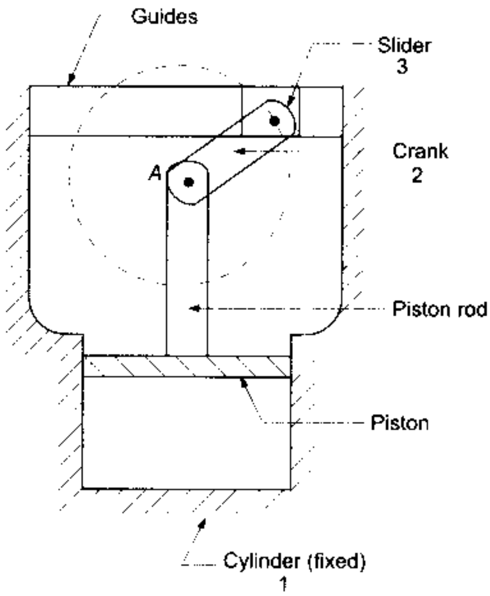


Fig.1.21 Donkey pump

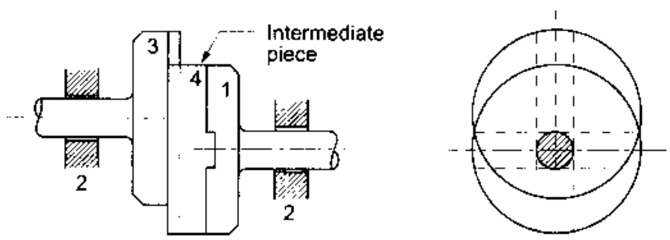


Fig.1.22 Oldham's coupling

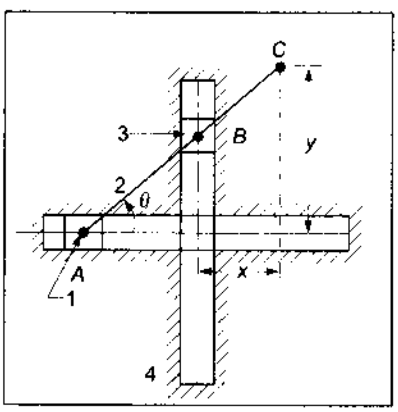


Fig.1.23 Elliptical trammel

$$x = BC\cos\theta \quad \text{or} \quad \frac{x}{BC} = \cos\theta$$

$$y = AC\sin\theta \quad \text{or} \quad \frac{y}{AC} = \sin\theta$$

Squaring and adding, we get

$$\frac{x^2}{BC^2} + \frac{y^2}{AC^2} = 1 \tag{1.15}$$

which is the equation of an ellipse.

Scotch yoke This mechanism gives simple harmonic motion. Its early application was on steam pumps, but it is now used as a mechanism on a test machine to produce vibrations. It is also used as a sine–cosine generator for computing elements. Fig.1.24 shows a sketch of scotch yoke mechanism.

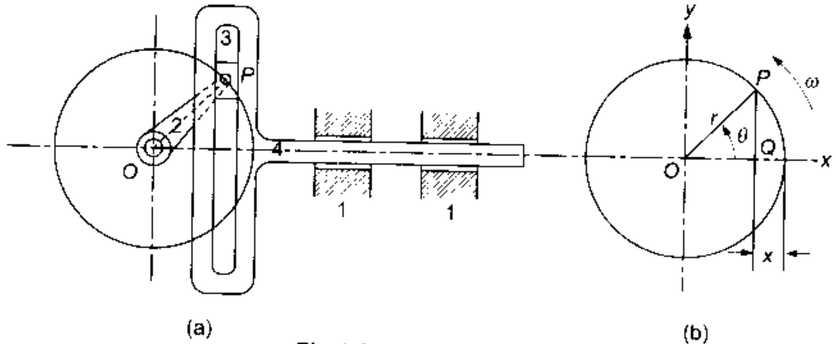


Fig.1.24 Scotch yoke

$$x = r - r\cos\theta$$

$$= r(1 - \cos\omega t) \tag{1.16}$$

$$v = \mathrm{d}x/\mathrm{d}t = r\omega\sin\omega t \tag{1.17}$$

$$a = \mathrm{d}^2x/\mathrm{d}t^2 = r\omega^2\cos\omega t \tag{1.18}$$

Example 1.1

Determine the type of chain in Fig.1.25(a)–(e).

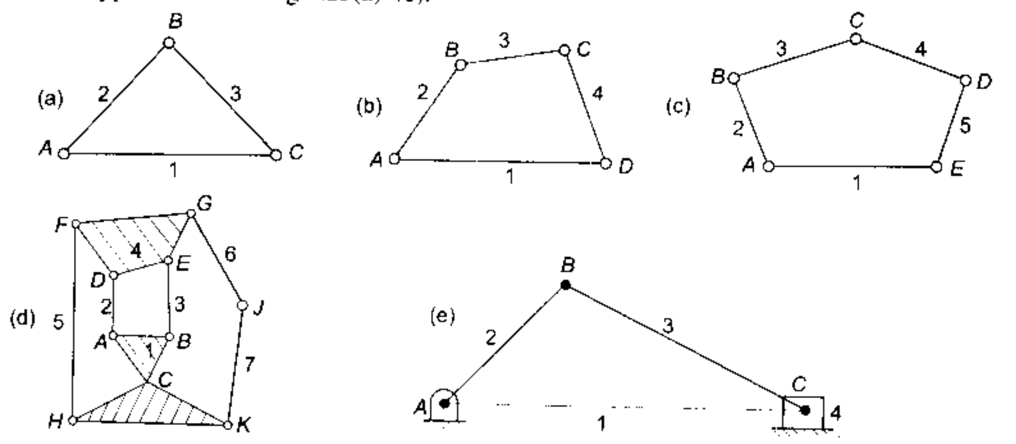


Fig.1.25 Type of chain

Solution

(a) $N = 2P - 4$ $\qquad\qquad$ $J = \frac{3N}{2} - 2$

$N = 3, P = 3, J = 3$

Case (i) LHS = 3
RHS = $2 \times 3 - 4 = 2$
LHS > RHS

Case (ii) LHS = 3
RHS = $\frac{3 \times 3}{2} - 2 = 3.5$
LHS > RHS

It is a locked chain and not a kinematic chain.

(b) $N = 4, P = 4, J = 4$

Case (i) LHS = 4
RHS = $2 \times 4 - 4 = 4$
LHS = RHS

Case (ii) LHS = 4
RHS = $\frac{3 \times 4}{2} - 2 = 4$
LHS = RHS

It is a constrained kinematic chain.

(c) $N = 5, P = 5, J = 5$

Case (i) LHS = 5
RHS = $2 \times 5 - 4 = 6$
LHS < RHS

Case (ii) LHS = 5
RHS = $\frac{3 \times 5}{2} - 2 = 5.5$
LHS < RHS

It is an unconstrained chain and not a kinematic chain.

(d) $N = 6, P = 5, J = 7$

Case (i) LHS = 6
RHS = $2 \times 5 - 4 = 6$
LHS = RHS

Case (ii) LHS = 7
RHS = $\frac{3 \times 6}{2} - 2 = 7$
LHS = RHS

It is a constrained kinematic chain.

(e) $N = 4, P = 4, J = 4$

Case (i) LHS = 4
RHS = $2 \times 4 - 4 = 4$
LHS = RHS

Case (ii) LHS = 4
RHS = $\frac{3 \times 4}{2} - 2 = 4$
LHS = RHS

It is a constrained kinematic chain.

Example 1.2

Determine the number of degrees of freedom of the mechanism shown in Fig.1.26(a)–(h).

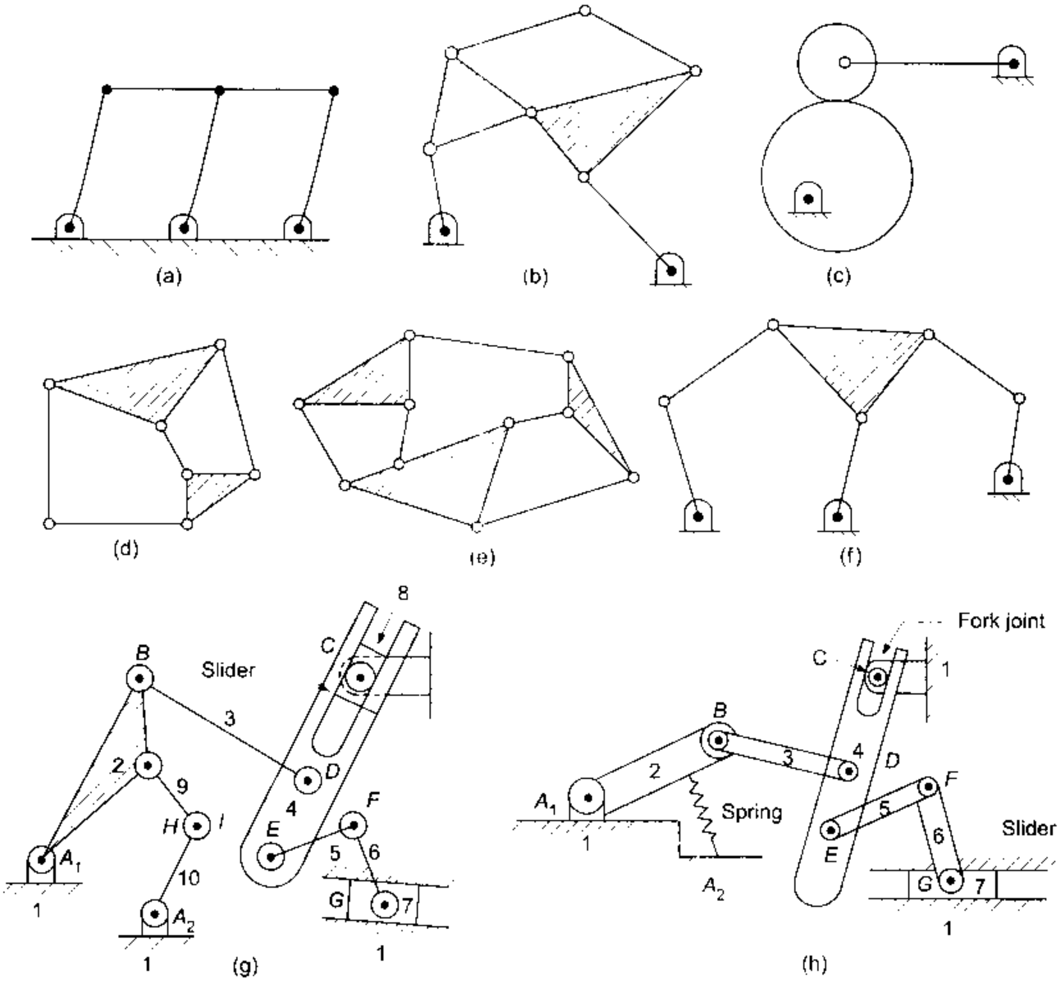


Fig.1.26 Degrees of freedom for different mechanisms

■ **Solution**

(a) $N = 5; \quad P_1 = 6$

$F = 3(5 - 1) - 2 \times 6 = 0$

(c) $N = 3; \quad P_1 = 2; \quad P_2 = 1$

$F = 3(3 - 1) - 2 \times 2 - 1 = 1$

(b) $N = 7; \quad N_2 = 5; \quad N_3 = 2$

$$P_1 = \frac{(2 \times 5 + 3 \times 2)}{2} = 8$$

$F = 3(7 - 1) - 2 \times 8 = 2$

(d) $N_2 = 4; \quad N_3 = 2$

$N = 4 + 2 = 6$

$$P_1 = \frac{(2 \times 4 + 3 \times 2)}{2} = 7$$

$F = 3(6 - 1) - 2 \times 7 = 1$

(e) $N_2 = 5;\quad N_3 = 2;\quad N_4 = 1$

$N = 5+2+1 = 8$

$P_1 = \dfrac{(2\times5+3\times2+4\times1)}{2} = 10$

$F = 3(8-1) - 2\times10 = 1$

(f) $N_2 = 6;\quad N_3 = 1$

$N = 6+1 = 7$

$P_1 = \dfrac{(2\times6+3\times1)}{2} = \dfrac{15}{2}$

$F = 3(7-1) - 2\times\dfrac{15}{2} = 3$

(g) $N = 10;\quad P_1 = 12$

$F = 3(10-1) - 2\times12 = 3$

(h) $N = 7;\quad P_1 = 7;\quad P_2 = 1$

$F = 3(7-1) - 2\times7 - 1\times1 = 3$

Example 1.3

Find the equivalent mechanisms with turning pairs for the mechanisms shown in Figs.1.27–1.30.

■ **Solution**

1. A spring can be replaced by two binary links. Therefore, the equivalent mechanism is as shown in Fig.1.27(b).

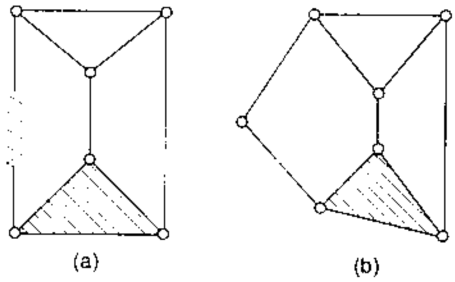


Fig.1.27 Equivalent mechanism for a cam pair

2. A cam pair can be replaced by one binary link with two turning pairs at each end. Therefore, the equivalent mechanism is shown in Fig.1.28(b). The centres of curvature at the point of contact *E* lie at *B* and *C*, respectively.

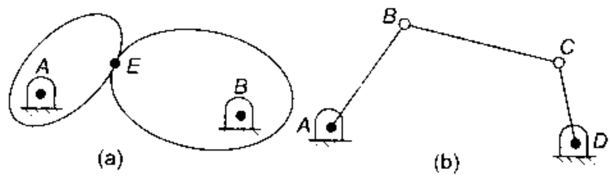


Fig.1.28 Equivalent mechanism

3. The equivalent mechanism is shown in Fig.1.29(b) as explained in item 2 above.

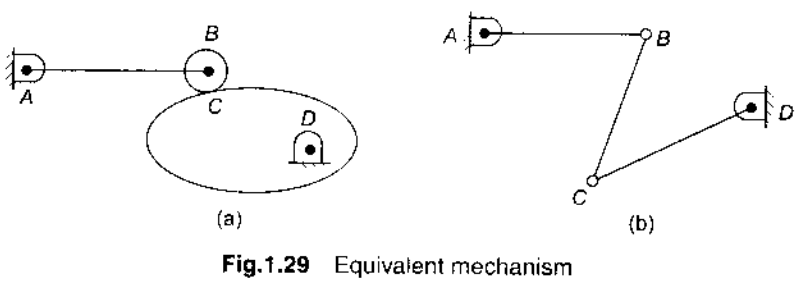


Fig.1.29 Equivalent mechanism

4. A spring is equivalent to two binary links connected by a turning pair. A cam follower is equivalent to one binary link with turning pairs at each end. The equivalent chain with turning pairs is shown in Fig.1.30(b).

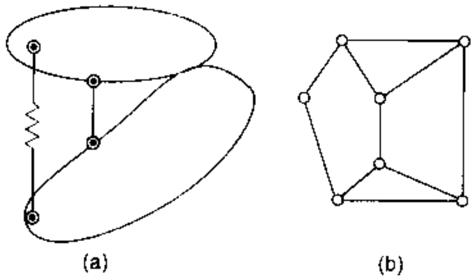


Fig.1.30 Equivalent mechanism for two binary links

Example 1.4

Determine the number of degrees of freedom of the mechanisms shown in Fig.1.31(a)–(f).

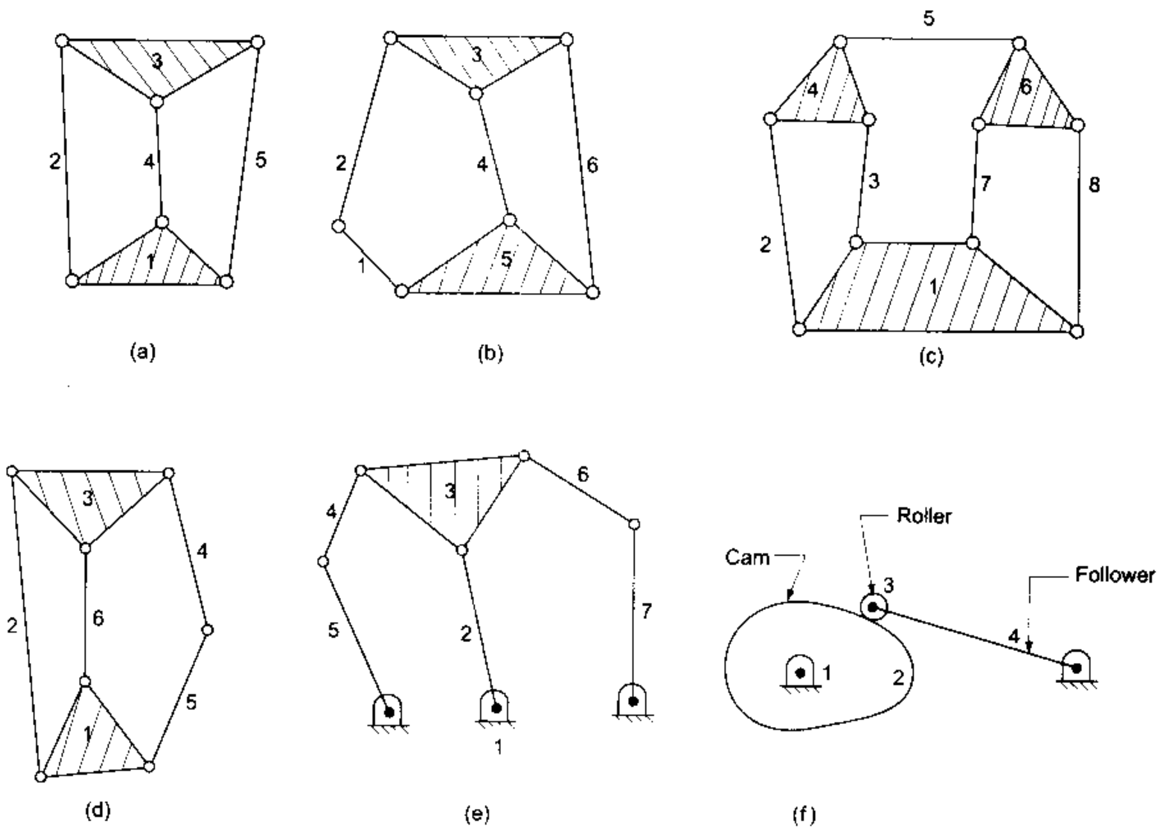


Fig.1.31 Number of degrees of freedom for the mechanisms

Solution

(a)
$$N = 5;\quad N_2 = 3;\quad N_3 = 2$$
$$P_1 = \frac{(2N_2 + 3N_3)}{2} = \frac{(2\times 3 + 3\times 2)}{2} = 6$$
$$F = 3(N-1) - 2P_1 = 3(5-1) - 2\times 6 = 0$$

(b)
$$N = 6;\quad N_2 = 4;\quad N_3 = 2$$
$$P_1 = \frac{(2N_2 + 3N_3)}{2} = \frac{(2\times 4 + 3\times 2)}{2} = 7$$
$$F = 3(N-1) - 2P_1 = 3(6-1) - 2\times 7 = 1$$

(c)
$$N = 8;\quad N_2 = 5;\quad N_3 = 2;\quad N_4 = 1$$
$$P_1 = \frac{(2N_2 + 3N_3 + 4N_4)}{2} = \frac{(2\times 5 + 3\times 2 + 4\times 1)}{2} = 10$$
$$F = 3(N-1) - 2\times P_1 = 3(8-1) - 2\times 10 = 1$$

(d)
$$N = 6;\quad N_2 = 4;\quad N_3 = 2$$
$$P_1 = \frac{(2N_2 + 3N_3)}{2} = \frac{(2\times 4 + 3\times 2)}{2} = 7$$
$$F = 3(N-1) - 2P_1 = 3(6-1) - 2\times 7 = 1$$

(e)
$$N = 6;\quad N_2 = 4;\quad N_3 = 2$$
$$P_1 = \frac{(2N_2 + 3N_3)}{2} = \frac{(2\times 4 + 3\times 2)}{2} = 7$$
$$F = 3(N-1) - 2P_1 = 3(6-1) - 2\times 7 = 1$$

(f)
$$N = 3;\quad P_1 = 2;\quad P_2 = 1$$
$$F = 3(N-1) - 2P_1 - P_2 = 3(3-1) - 2\times 2 - 1 = 1$$

Example 1.5

In a crank and slotted lever quick-return mechanism shown in Fig.1.32, the distance between the fixed centres is 250 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted lever with the vertical in the extreme position and the ratio of time of cutting stroke to return stroke.

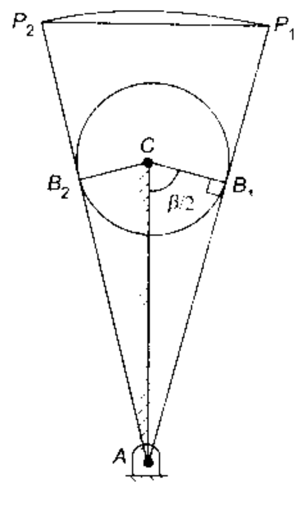


Fig.1.32 Crank and slotted lever quick-return mechanism

Solution

$$AC = 250 \text{ mm}, CB_1 = 120 \text{ mm}$$

$$\cos(\beta/2) = \frac{B_1C}{AC}$$

$$= \frac{120}{250} = 0.48$$

$$\beta = 123.63°$$

$$\alpha = 360° - 123.63° = 237.37°$$

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta}$$

$$= \frac{237.37°}{123.63°} = 1.9356$$

$$\text{Inclination of slotted lever with the vertical} = 90° - \frac{\beta}{2}$$

$$= 28.68°$$

Example 1.6

In a Whitworth quick-return motion mechanism, as shown in Fig.1.33, the distance between the fixed centres is 60 mm and the length of the driving crank is 80 mm. The length of the slotted lever is 160 mm and the length of the connecting rod is 140 mm. Calculate the ratio of the time of cutting to return strokes.

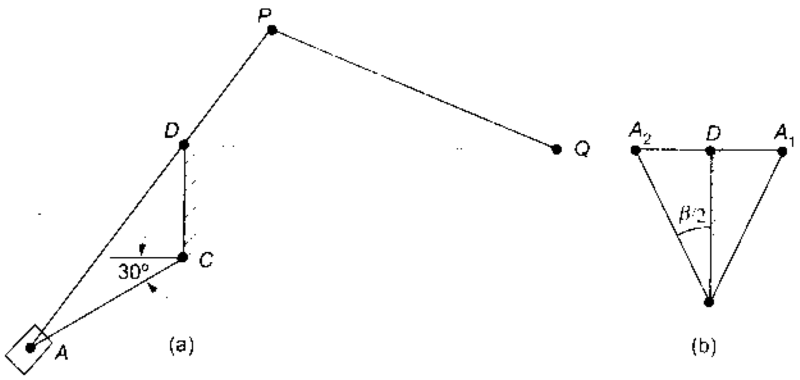


Fig.1.33 Whitworth quick-return motion mechanism

Solution

$$CD = 60 \text{ mm}, CA = 80 \text{ mm}, PA = 160 \text{ mm}, PQ = 140 \text{ mm}$$

$$\cos(\beta/2) = \frac{CD}{CA_2} = \frac{60}{80} = 0.75$$

$$\beta = 83.82°$$

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{(360° - \beta)}{\beta}$$

$$= \frac{277.18°}{83.82°} = 3.347$$

Example 1.7

The distance between two parallel shafts connected by Oldham's coupling is 20 mm. The driving shaft revolves at 200 rpm. Determine the maximum speed of sliding of the tongue of the intermediate piece along its groove.

Solution

$$d = 20 \text{ mm}, n = 200 \text{ rpm}$$

$$\omega = 2\pi \times \frac{200}{60} = 20.94 \text{ rad/s}$$

$$\text{Maximum velocity of sliding} = \omega \times d = 20.94 \times 0.020 = 0.419 \text{ m/s}$$

Example 1.8

In a crank and slotted lever mechanism, the length of the crank is 500 mm and the ratio of time of cutting stroke to return stroke is 3.25. Determine (a) the distance between the fixed centres and (b) the length of the slotted lever, if the length of the stroke is 200 mm.

Solution

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = 3.25$$

$$\frac{360° - \beta}{\beta} = 3.25$$

$$\beta = 110.77°$$

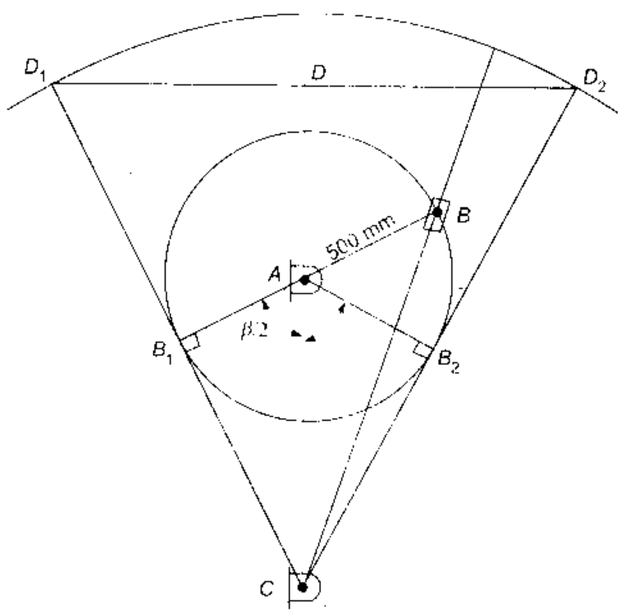


Fig.1.34 Crank and slotted lever

From Fig.1.34, we have

$$\frac{AB_1}{AC} = \cos\left(\frac{\beta}{2}\right)$$

$$AC = \frac{500}{\cos 55.38^\circ} = 880.2 \text{ mm}$$

$$\text{Length of stroke} = D_1D_2 = 2D_1D$$

$$200 = 2CD_1 \sin\left(90^\circ - \frac{\beta}{2}\right)$$

$$= 2CD_1 \sin 34.62^\circ$$

$$\text{Length of slotted lever, } CD_1 = 176 \text{ mm}$$

Example 1.9

The configuration of a drag link mechanism is shown in Fig.1.35. Determine the time ratio and the length of stroke, if the crank O_2A rotates clockwise.

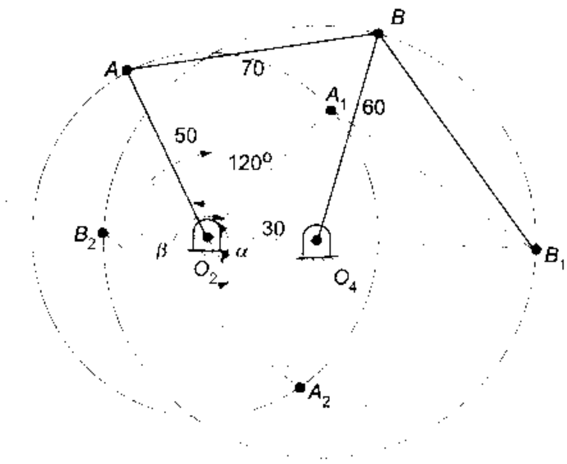


Fig.1.35 Drag link mechanism

■ Solution

The drag link mechanism has been drawn as O_1ABO_4 to a scale of 1 cm = 10 mm. The extreme positions of B are B_1 and B_2. The length of stroke is $2 \times O_4B = B_1B_2 = 2 \times 60 = 120$ mm.

When B is at B_1, then A is at A_1 and when B is at B_2, then A is at A_2.

By measurement, we have

$$\alpha = 110^\circ \text{ and } \beta = 250^\circ$$

$$\text{Ratio of times} = \frac{\beta}{\alpha} = \frac{250}{110} = 3.27$$

Exercises

1 Calculate the number of degrees of freedom of the mechanisms shown in Fig.1.36(a–e).

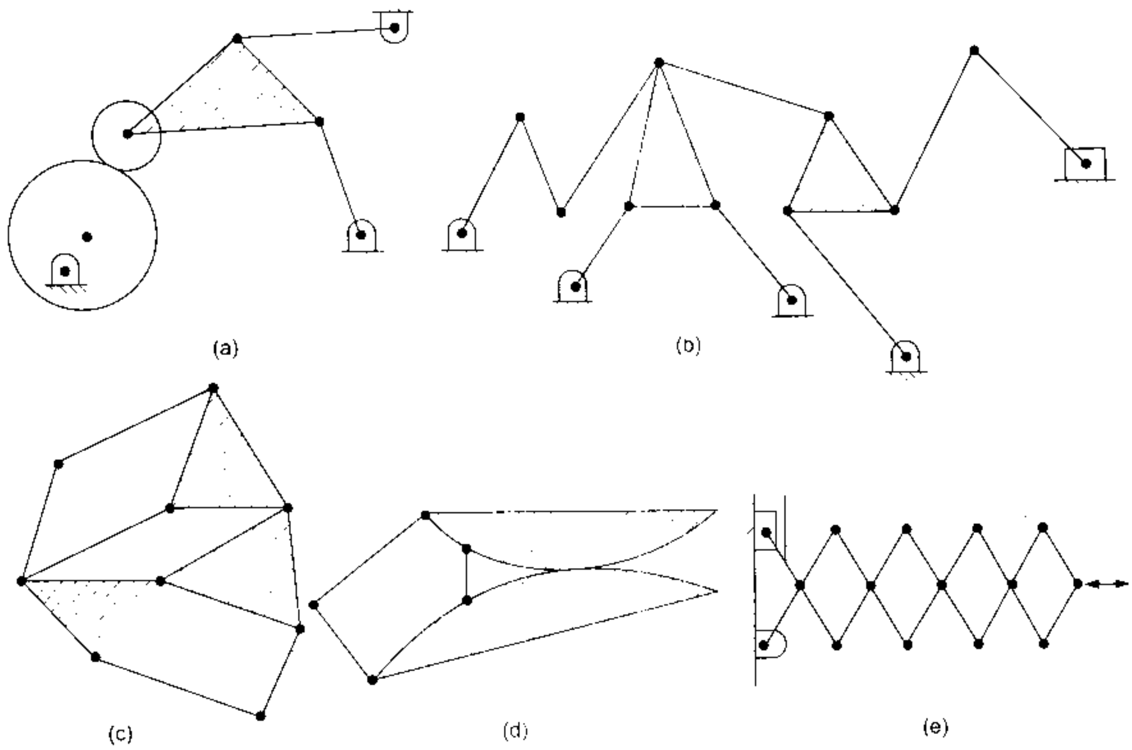


Fig.1.36 Degrees of freedom for different mechanisms

2 Explain the following:

(a) Inversion of mechanisms

(b) The principles and applications of the inversions of the slider–crank chain.

3 What do you understand by inversion of a chain? Describe the mechanisms obtained by inversion of the four-bar chain.

4 (a) What is meant by inversion? Describe the possible inversions obtained out of double slider–crank chain. (b) Distinguish between higher and lower pairs with examples.

5 Describe the three kinds of lower pairs, giving a sketch of each kind, and state the types of relative motion that each pair permits.

6 Show the various inversions of a kinematic chain with four binary links having two revolute and two prismatic pairs. How do the relative dimensions of the links determine the movability of a four-bar linkage?

7 Prove that the minimum number of binary links in a constrained mechanism with simple hinges is four.

8 What do you understand by the term degrees of freedom? For a planar mechanism, derive the expression for Gruebler's criterion.

9 What do you understand by constrained motion? What are the different types of constrained motions? Explain each type with examples.

10 Explain the following mechanisms: (a) Elliptical trammel (b) Oldham's coupling (c) Scotch yoke (d) Donkey pump

11 In a crank and slotted lever quick-return mechanism, the distance between the fixed centres is 150 mm and the driving crank is 100 mm long. Find the ratio of the time taken during the cutting and return strokes.

12 The distance between the axes of parallel shafts connected by Oldham's coupling is 25 mm, the speed of rotation of the shafts is 320 rpm. Determine the maximum velocity of sliding of each tongue in its slot.

13 Design a quick-return mechanism of the type shown in Fig.1.37. The working stroke is 200 mm and the ratio of the time of working stroke to return stroke is 2:1. The driving crank is 50 mm long.

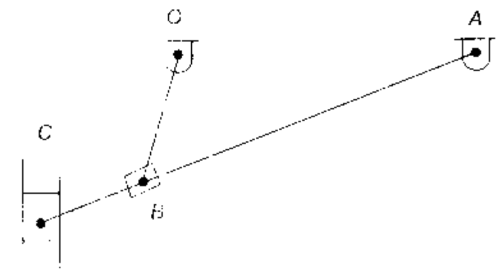


Fig.1.37 Quick-return mechanism

14 Design a Whitworth quick-return mechanism shown in Fig.1.38 to have the following particulars:

Return stroke = 200 mm, time ratio of working to return stroke = 2, length of driving crank = 50 mm.

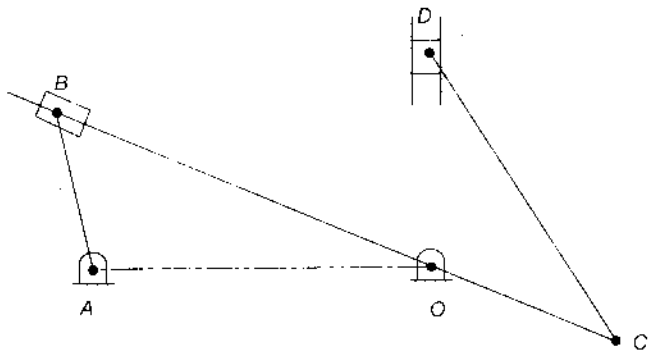


Fig.1.38 Whitworth quick-return mechanism

15 A drag link quick-return mechanism is shown in Fig.1.39. Determine the time ratio of the working stroke to the return stroke for uniform angular velocity of crank O_1A.

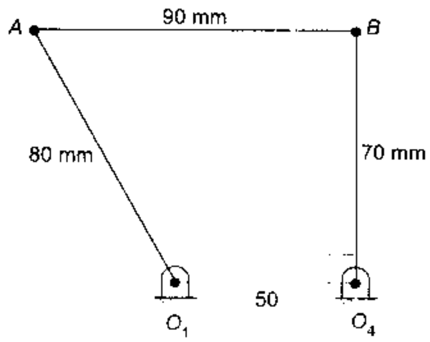
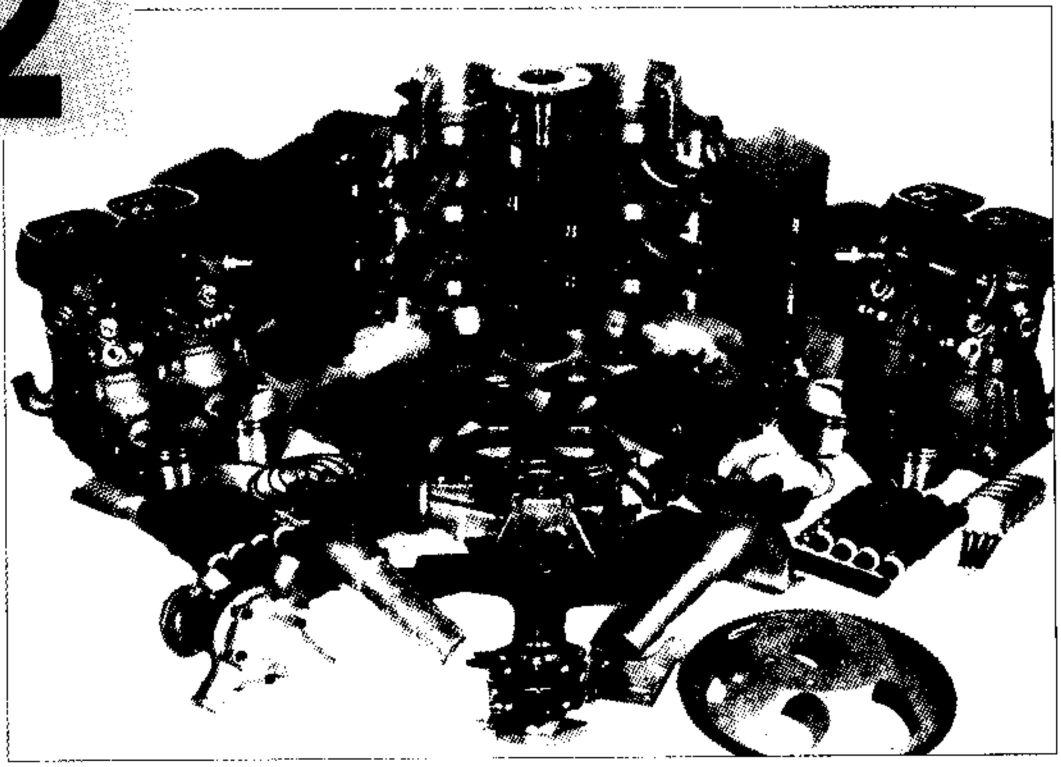


Fig.1.39 Drag link quick-return mechanism

2



KINEMATICS

2.1 INTRODUCTION

Kinematics deals with the study of relative motion between the various links of a machine, ignoring the forces involved in producing that motion. Thus, kinematics is the study to determine the displacement, velocity and acceleration of the various links of a mechanism. A machine is a mechanism or a combination of mechanisms that not only imparts definite motion to the various links but also transmits and modifies the available mechanical energy to some kind of useful energy. In this chapter, we shall study the various methods used to determine the velocity and acceleration in mechanisms.

2.2 VELOCITY DIAGRAMS

Displacement The displacement of a body is its change of position with reference to a certain fixed point.

Velocity This is the state of change of displacement of a body with respect to time. This is a vector quantity.

Linear velocity This is the rate of change of displacement of a body along a straight line with respect to time. Its units are m/s.

Angular velocity This is the rate of change of angular position of a body with respect to time. Its units are rad/s.

The relationship between velocity v and angular velocity ω is:

$$v = r\omega \tag{2.1}$$

where r = distance of point undergoing displacement from the centre of rotation.

Relative velocity The relative velocity of a body A with respect to a body B is obtained by adding the reversed velocity of B to the velocity of A. If $v_a > v_b$, then

$$v_{ab} = v_a - v_b$$

or

$$ba = oa - ob$$

Similarly,

$$v_{ba} = v_b - v_a$$

or

$$ab = ob - oa$$

Consider two points A and B on a rigid link rotating clockwise about A, as shown in Fig.2.1(a). There can be no relative motion between A and B as long as the distance between them remains the same. Therefore, the relative motion of B with respect to A must be perpendicular to AB. Hence, the direction of relative velocity of two points in a rigid link with respect to each other is always along the perpendicular to the straight line joining them. Let relative velocity of B with respect to A be represented by $v_{ba} = \omega \cdot AB$, then ab is drawn perpendicular to AB to a convenient scale, as shown in Fig.2.1(b). Similarly, the linear velocity of any other point C on AB with respect to A is $v_{ca} = \omega \cdot AC$ and is represented by vector ac. Hence,

$$\frac{v_{ba}}{v_{ca}} = \frac{\omega \cdot AB}{\omega \cdot AC} = \frac{AB}{AC}$$

or

$$\frac{ab}{ac} = \frac{AB}{AC} = \frac{r}{r_1} \tag{2.2}$$

Hence the point c divides the vector ab in the same ratio as the point C divides the link AB.

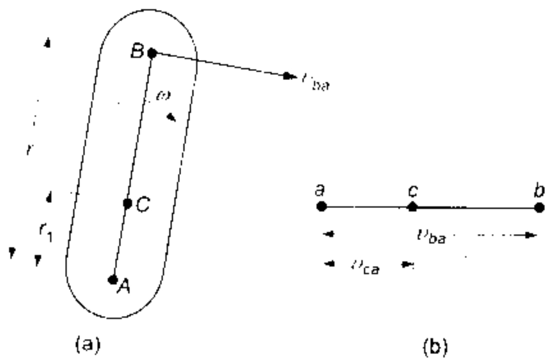


Fig.2.1 Relative velocity of a point

2.3 DETERMINATION OF LINK VELOCITIES

There are two methods to determine the velocities of links of mechanisms:

1. Relative velocity method 2. Instantaneous centre method

2.3.1 Relative Velocity Method

Consider a rigid link *AB*, as shown in Fig.2.2(a), such that the velocity of *A* (v_a) is vertical and the velocity of *B* (v_b) is horizontal. To construct the velocity diagram, take a point *o*. Draw *oa* representing the magnitude and direction of the velocity of *A*. Draw *ob* along the direction of v_b. From point *a* draw a line *ab* perpendicular to *AB*, meeting *ob* in *b*. Then *oab* is the velocity triangle, as shown in Fig.2.2(b). $ob = v_b$; $ab = v_{ba}$, that is, the velocity of *B* with respect to *A*. Vector *ab* is called the *velocity image* of link *AB*. The velocity of any point *C* in *AB* with respect to *A* is given by,

$$v_{ca} = ac = v_{ba} \cdot \left(\frac{AC}{AB}\right)$$

$$v_c = v_a + \left(\frac{AC}{AB}\right) v_{ba}$$

$$= oa + \left(\frac{ac}{ab}\right) \cdot ab$$

$$= oa + ac = oc$$

Hence vector *oc* represents the velocity of point *C*.

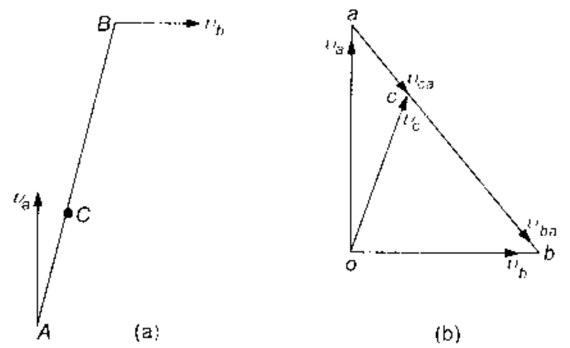


Fig.2.2 Relative velocity of points in a kinematic link

2.3.2 Relative Velocity of Points in a Kinematic Link

Consider a link A_1B_1 first undergoing rotation by an amount $\Delta\theta$ to $A_1B'_1$ and then undergoing translation by an amount Δs_A to occupy the new position A_2B_2, as shown in Fig.2.3(a). Then

$$\Delta s_B = \Delta s_{BA} + \Delta s_A \tag{2.3}$$

Now let the link A_1B_1 first undergo linear translation Δs_A to $A_1B''_1$ and then angular rotation $\Delta\theta$ to A_2B_2, as shown in Fig.2.3(b). Then.

$$\Delta s_B = \Delta s_A + \Delta s_{BA} \tag{2.4}$$

Equations (2.3) and (2.4) are same. Dividing by Δt, we get

$$\frac{\Delta s_B}{\Delta t} = \frac{\Delta s_A}{\Delta t} + \frac{\Delta s_{BA}}{\Delta t}$$

or

$$v_b = v_a + v_{ba}$$

Therefore, the velocity of point B is obtained by adding vectorially the relative velocity of point B with respect to point A to the velocity of point A.

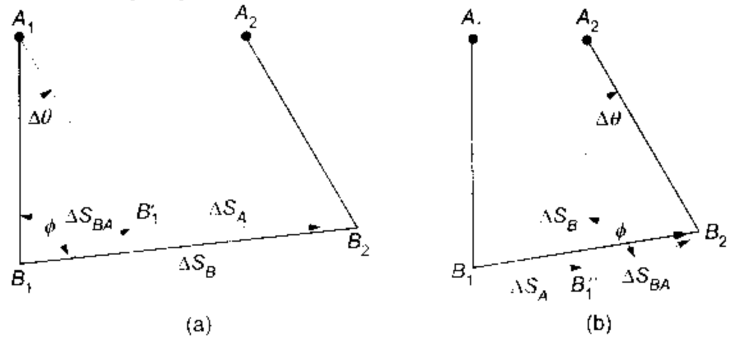


Fig.2.3 Relative velocity of points in a kinematic link

Now,
$$\Delta s_{BA} = A_1B_1 \cdot \Delta\theta$$
$$\frac{\Delta s_{BA}}{\Delta t} = A_1B_1 \cdot \frac{\Delta\theta}{\Delta t}$$
or
$$v_{ba} = A_1B_1 \cdot \omega$$
Also,
$$\angle\phi = 90^\circ$$

The following conclusions may be drawn from the above analysis:

- The velocity of any point on the kinematic link is given by the vector sum of the velocity of some other point in the link and the velocity of the first point relative to the other.
- The magnitude of the velocity of any point on the kinematic link relative to the other point in the kinematic link is the product of the angular velocity of the link and distance between the two points under consideration.
- The direction of the velocity of any point on a link relative to any other point on the link is perpendicular to the line joining the two points.

2.3.3 Relative Angular Velocities

Consider two links OA and OB connected by a pin joint at O, as shown in Fig.2.4. Let ω_1 and ω_2 be the angular velocities of the links OA and OB, respectively. Relative angular velocity of OA with respect to OB is,

$$\omega_{12} = \omega_1 - \omega_2$$

and relative angular velocity of OB with respect to OA is,

$$\omega_{21} = \omega_2 - \omega_1$$
$$= -\omega_{12}$$

If r = radius of the pin at joint O, then Rubbing velocity at the pin joint O

$= (\omega_1 - \omega_2)\,r$, when the links move in the same direction (2.5a)

$= (\omega_1 + \omega_2)\,r$, when the links move in the opposite direction. (2.5b)

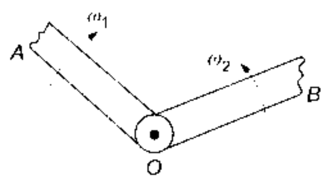


Fig.2.4 Relative angular velocities

2.3.4 Relative Velocity of Points on the Same Link

Consider a ternary link *ABC*, as shown in Fig.2.5(a), such that *C* is any point on the link. Let v_a and v_b be the velocities of points *A* and *B*, respectively. Then

$$v_b = v_a + v_{ba}$$

$$\omega_{ab} = \frac{v_{ba}}{AB} = \frac{ab}{AB}$$

$$v_c = v_a + v_{ca}$$

or

$$= v_b + v_{cb}$$

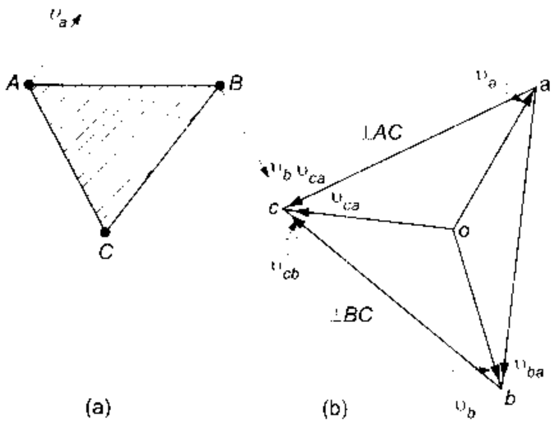


Fig.2.5 Relative velocity of points on the same link

The velocity diagram is shown in Fig.2.5(b).
Angular velocity of link *ABC*,

$$\omega_{abc} = \frac{v_{ba}}{AB} = \frac{v_{cb}}{BC} = \frac{v_{ca}}{AC}$$

$$= \frac{ab}{AB} = \frac{bc}{BC} = \frac{ac}{AC} \tag{2.6}$$

2.3.5 Forces in a Mechanism

Consider a link *AB* subjected to the action of forces and velocities, as shown in Fig.2.6. Let *A* be the driving end and *B* the driven end. When the direction of the forces and velocities is the same, then

$$\text{Energy at } A = \text{Energy at } B$$

$$F_a \times v_a = F_b \times v_b$$

or

$$F_b = \frac{F_a v_a}{v_b} \tag{2.7a}$$

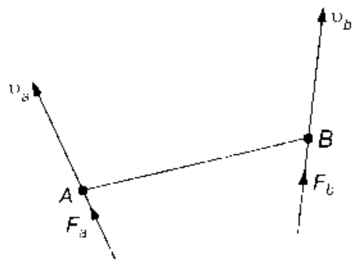


Fig.2.6 Force and velocity diagram

Considering the effect of friction, the efficiency of transmission,

$$\eta = \frac{\text{output}}{\text{input}} = \frac{(F_b v_b)}{(F_a v_a)}$$

or

$$F_b = \frac{\eta F_a v_a}{v_b} \qquad (2.7b)$$

When the forces are not in the direction of the velocities, then their components along the velocities should be taken.

2.3.6 Mechanical Advantage

Mechanical advantage, $\text{MA} = \text{Load lifted / Effort applied}$

$$= \frac{F_b}{F_a}$$

For a mechanism, $\text{MA} = \dfrac{\text{Output torque}}{\text{Input torque}}$

$$= \frac{T_b}{T_a} = \frac{\omega_a}{\omega_b} \qquad (2.8a)$$

Considering the effect of friction, $\text{MA} = \eta \dfrac{\omega_a}{\omega_b} \qquad (2.8b)$

2.3.7 Four-bar Mechanism

(a) Consider the four-bar mechanism, as shown in Fig.2.7(a), in which the crank O_1A is rotating clockwise with uniform angular speed ω. The linear velocity of point A is $v_a = \omega\, O_1A$ and it is perpendicular to O_1A. Therefore, draw $o_1a \perp O_1A$ to a convenient scale in Fig.2.7(b). The velocity of point B is perpendicular to O_2B. Therefore, at point o_1, draw $o_1b \perp O_2B$. The relative velocity of B with respect to A is perpendicular to AB. Therefore, draw $ab \perp AB$ meeting the line drawn perpendicular to O_2B at b. Then $v_b = o_1b$, and $v_{ba} = ab$. To find the velocity of joint C, draw $ac \perp AC$ and $bc \perp BC$ to meet at c. Join o_1c. Then $v_c = o_1c$.

Now $\qquad v_b = v_a + v_{ba} = o_1a + ab = o_1b$

$\qquad\qquad v_c = v_b + v_{cb} = o_1b + bc = o_1c$

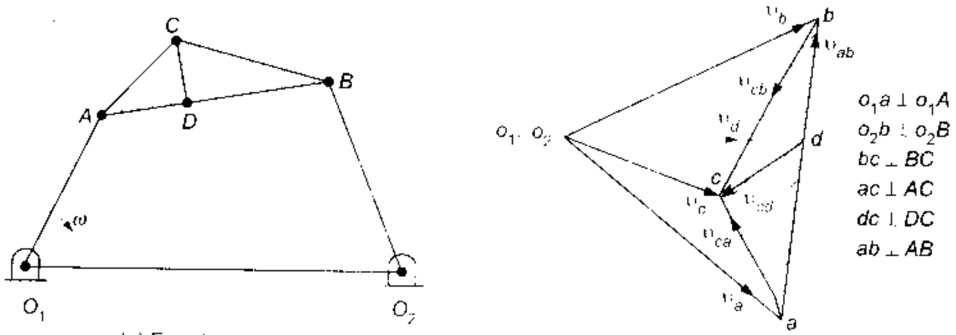
and $\qquad\quad\; = v_a + v_{ca} = o_1a + ac = o_1c$

To find the velocity of any point D in AB, we have

$$\frac{BD}{BA} = \frac{bd}{ba}$$

or

$$bd = \left(\frac{BD}{BA}\right) \cdot ba$$



(a) Four-bar mechanism (b) Velocity diagram

Fig.2.7

Thus, locate point d in ab and join o_1d. Then $o_1d = v_d$. To find the relative velocity of C with respect to D, join cd. Then $v_{cd} = dc$. The velocity image of link ABC is given by abc.

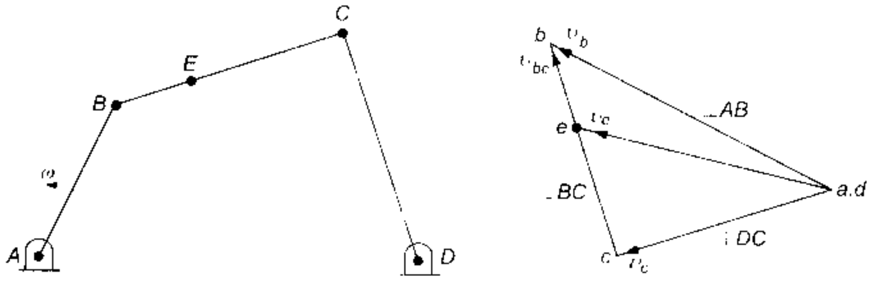
(b) Now consider the four-bar mechanism, as shown in Fig.2.8(a), in which the crank AB is rotating at angular velocity ω in the counter-clockwise direction. The absolute linear velocity of B is $\omega \cdot AB$ and is perpendicular to AB. Draw $ab \perp AB$ to a convenient scale to represent v_b, as shown in Fig.2.8(b). From b draw a line perpendicular to BC and from a another line perpendicular to CD to meet each other at point c. Then, $ac = v_c$ and $cb = v_{bc}$. To find the velocity of any point E in BC, we have

$$\frac{CE}{CB} = \frac{ce}{cb}$$

or

$$ce = \left(\frac{CE}{CB}\right) \cdot cb$$

Thus, locate point e in cb and join ae. Then $ae = v_e$.



(a) Four-bar mechanism (b) Velocity diagram

Fig.2.8

The rubbing velocities of points A–D are as follows:

Point A : $\omega_{ab} \cdot r_a$

Point B : $(\omega_{ab} \pm \omega_{cb})\, r_b$

Point C : $(\omega_{bc} + \omega_{dc})\, r_c$

Point D : $\omega_{cd} \cdot r_d$

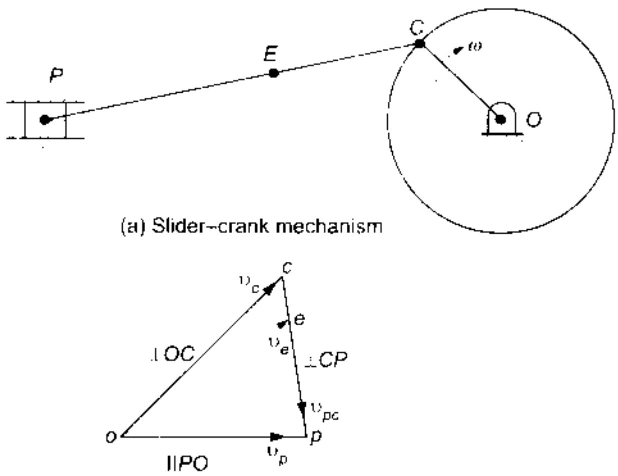
where r is the radius of the pin.

Use the +ve sign when angular velocities are in the opposite directions.

$$\omega_{cb} = \frac{bc}{BC} \quad \text{and} \quad \omega_{cd} = \frac{cd}{CD}$$

2.3.8 Slider–crank Mechanism

Consider the slider–crank mechanism, as shown in Fig.2.9(a), in which the crank OC is rotating clockwise with angular speed ω. PC is the connecting rod and P is the slider or piston.



(b) Velocity diagram

Fig.2.9

The linear velocity of C, $v_c = OC \cdot \omega$.

To draw the velocity diagram, draw a line $oc = v_c$ from point o, as shown in Fig.2.9(b), representing the velocity of point C to a convenient scale. From point c draw a line perpendicular to CP. The velocity of slider P is horizontal. Therefore, from point o draw a line parallel to OP to intersect the line drawn perpendicular to CP at p. Then the velocity of the piston, $v_p = op$ and the velocity of piston P relative to crank pin C is $v_{pc} = cp$. To find the velocity of any point E in CP, we have

$$\frac{CE}{CP} = \frac{ce}{cp}$$

or

$$ce = \left(\frac{CE}{CP}\right) \cdot cp$$

Thus locate point e in cp, join oe. Then $v_e = oe$.

The rubbing velocities at different points are as follows:

at $O : \omega_{oc} \cdot r_o$

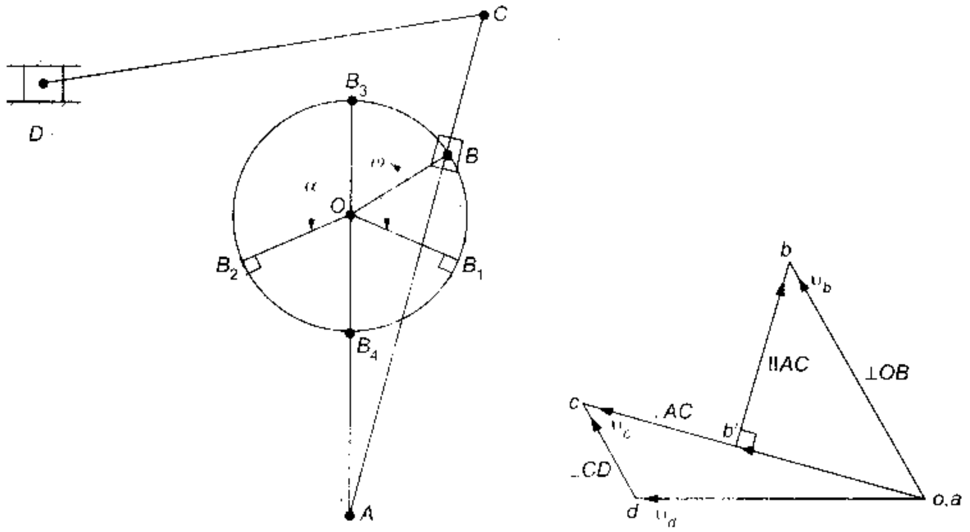
at $P : \omega_{cp} \cdot r_p$

at $C : (\omega_{oc} + \omega_{cp}) \cdot r_c$

2.3.9 Crank and Slotted Lever Mechanism

Consider AB the crank and slotted lever mechanism, as shown in Fig.2.10(a). The crank OB is rotating at uniform angular speed ω. Let $OB = r$, $AC = l$, and $OA = d$. Linear velocity of B, $v_b = r\omega$. Draw $ob = v_b$ and perpendicular to OB to a convenient scale, as shown in Fig.2.10(b). The velocity of point B on the crank OB is given by v_b. The velocity of the slotted lever is perpendicular to AC. The velocity of the slider is along the slotted lever. Hence draw a line from b parallel to AC to meet the line perpendicular to AC at b'.

Now
$$v_c = \left(\frac{AC}{AB}\right) \cdot ab'$$
$$= ac$$



(a) Crank and slotted lever mechanism (b) Velocity diagram

Fig.2.10

From point c draw a line perpendicular to CD and from point o draw a line parallel to the tool motion, to intersect at point d.

Velocity of cutting tool, $v_d = ad$

The component of the velocity of the crank perpendicular to the slotted lever is zero at positions B_1 and B_2. Thus for these positions of the crank, the slotted lever reverses its direction of motion.

$$\text{Time of cutting stroke / Time of return stroke} = \frac{\alpha}{(360^\circ - \alpha)}$$

At positions B_3 and B_4, the component of velocity along the lever is zero, that is, the velocity of the slider at the crank pin is zero. Thus the velocity of the lever at the crankpin is equal to the velocity of the crankpin, that is, $r\omega$. The velocities of the lever and tool at these points are at a minimum. The maximum cutting velocity occurs at B_3 and the maximum return velocity occurs at B_4.

$$\text{Maximum cutting velocity} = (OB \cdot \omega)\left(\frac{AC}{AB_3}\right)$$

$$= (r\omega)\left[\frac{l}{d+r}\right] \quad (2.9a)$$

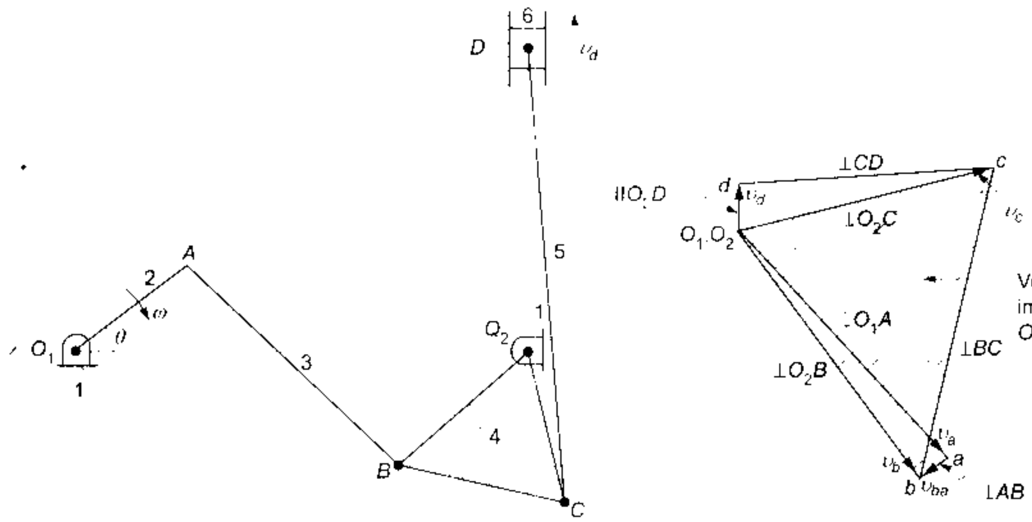
$$\text{Minimum return velocity} = (OB \cdot \omega)\left(\frac{AC}{AB_4}\right)$$

$$= (r\omega)\left[\frac{l}{d-r}\right] \quad (2.9b)$$

$$\text{Maximum cutting velocity / Maximum return velocity} = \left[\frac{d-r}{d+r}\right] \quad (2.10)$$

2.3.10 Drag Mechanism

The drag mechanism is shown in Fig.2.11(a). Link 2 rotates at constant angular speed ω. Link 4 rotates at a non-uniform velocity. Ram 6 will move with nearly constant velocity over most of the upward stroke to give a slow upward stroke and a quick downward stroke when link 2 rotates clockwise.



(a) Drag mechanism (b) Velocity diagram

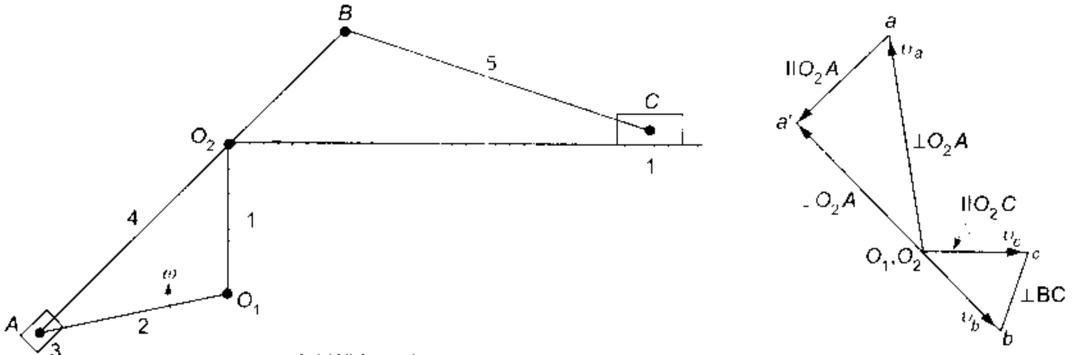
Fig.2.11

To determine the velocity diagram, draw $o_1a = \omega \cdot O_1A$, perpendicular to O_1A, as shown in Fig.2.11(b). From a draw $ab \perp AB$ and from o_1 draw $o_1b \perp O_2B$ to intersect at b. Then, $o_1b = v_b$ and $ab = v_{ba}$. At b draw $bc \perp BC$ and at o_1 draw $o_1c \perp O_2C$, to intersect at c. Then $o_1c = v_c$.

Now at o_1 draw $o_1d \parallel O_2D$ and from c draw a line perpendicular to CD to intersect at d. Then $o_1d = v_d$, the velocity of ram 6. The velocity image of O_2BC is o_2bc.

2.3.11 Whitworth Quick-return Mechanism

In the Whitworth mechanism, as shown in Fig.2.12(a), the crank O_1A rotates with constant angular speed ω. The link AB oscillates about pin O_2. The ram C reciprocates on the guides.



(a) Whitworth mechanism
(b) Velocity diagram

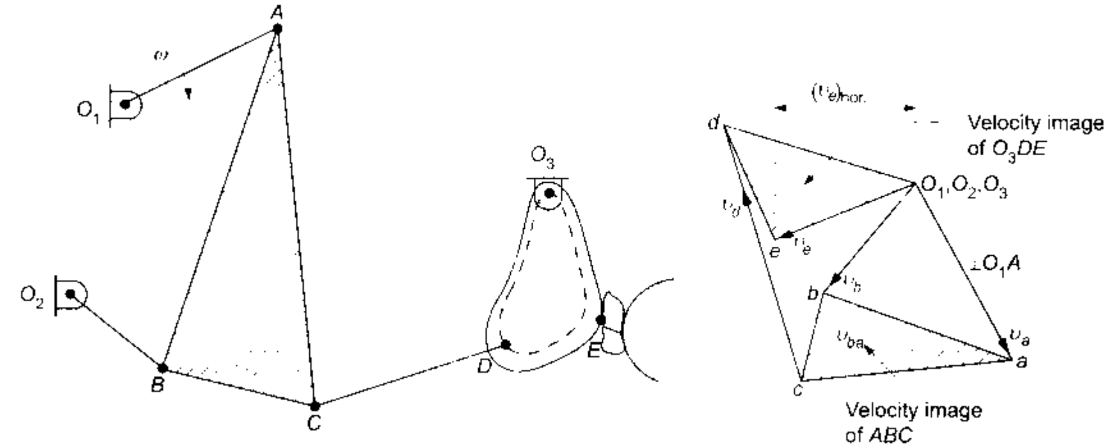
Fig.2.12 Whitworth quick-return mechanism

The velocity diagram has been drawn in Fig.2.12(b). $v_a = o_1a = \omega \cdot O_1A$.

Draw $o_1a \perp O_1A$ to a convenient scale. At o_1 draw a line perpendicular to AB and at a draw a line parallel to O_2A to intersect at a'. Then aa' represents the velocity of slider at A and o_1a' the velocity of lever AB at A. Produce $a'o_1$ to b so that $a'o_1/o_1b = O_2A/O_2B$. Draw $bc \perp BC$ and $o_1c \parallel O_2C$ to intersect at c. Then $v_c = o_1c$ is the velocity of the ram.

2.3.12 Stone Crusher Mechanism

In the stone crusher mechanism shown in Fig.2.13(a), the crank O_1A rotates at uniform angular speed ω. To obtain the velocity diagram [Fig.2.13(b)], draw $o_1a = v_a = \omega \cdot O_1A$ perpendicular to O_1A to a convenient scale. At o_1 draw a line perpendicular to O_2B and at a another line perpendicular to AB to intersect at b. Then $o_1b = v_b$ and $ab = v_{ba}$. At b draw a line perpendicular to BC and at a one perpendicular to AC to intersect at c. At c draw a line perpendicular to CD and at o_1 one perpendicular to O_3D to intersect at d. Then $cd = v_d$. At d draw $de \perp DE$ and $o_1e \perp O_3E$. Then $o_1e = v_e$. The horizontal component of $v_e = (v_e)_{hor}$.



(a) Stone crusher mechanism
(b) Velocity diagram

Fig.2.13

Let F be the horizontal force to be overcome and T be the torque needed at the driving crank. Then

$$T\omega = F(v_c)_{hor}$$

or

$$T = \frac{F(v_c)_{hor}}{\omega} \tag{2.11}$$

Example 2.1

In the mechanism shown in Fig.2.14(a), the crank O_1A rotates at a uniform speed of 650 rpm. Determine the linear velocity of the slider C and the angular speed of the link BC. $O_1A = 30$ mm, $AB = 45$ mm, $BC = 50$ mm, $O_2B = 65$ mm, and $O_1O_2 = 70$ mm.

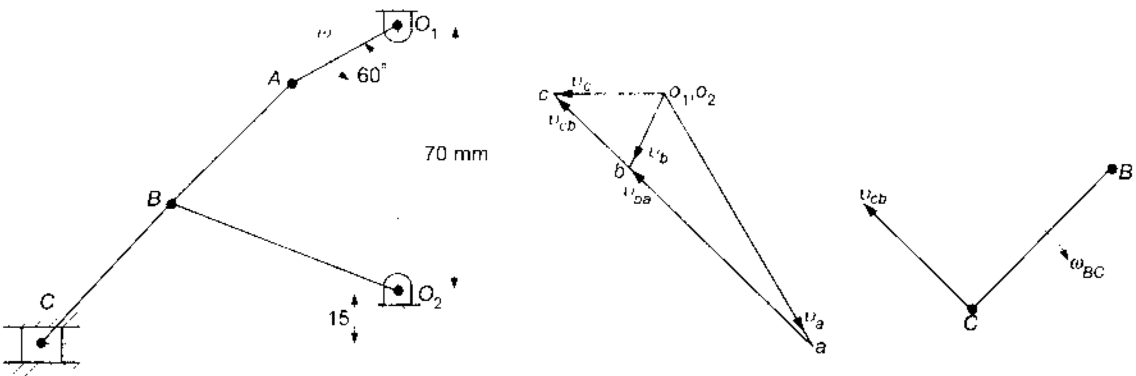


Fig.2.14

Solution

$$\omega = 2\pi \times \frac{650}{60} = 68.07 \text{ rad/s}$$
$$v_a = \omega \cdot O_1A = 68.07 \times 0.03 = 2.04 \text{ m/s}$$

Draw $o_1a \perp O_1A$ to a scale of 1 cm = 0.5 m/s. Draw a line from o_1 perpendicular to O_2B and another line from a perpendicular to AB to intersect at b. Then $ab = v_{ba}$ and $o_1b = v_b$.

From b draw a line perpendicular to BC and another line from o_1 parallel to the path of motion of the slider to intersect at c. Then $o_1c = v_c$, the velocity of the slider. The velocity diagram is shown in Fig.2.14(b). By measurement, we get

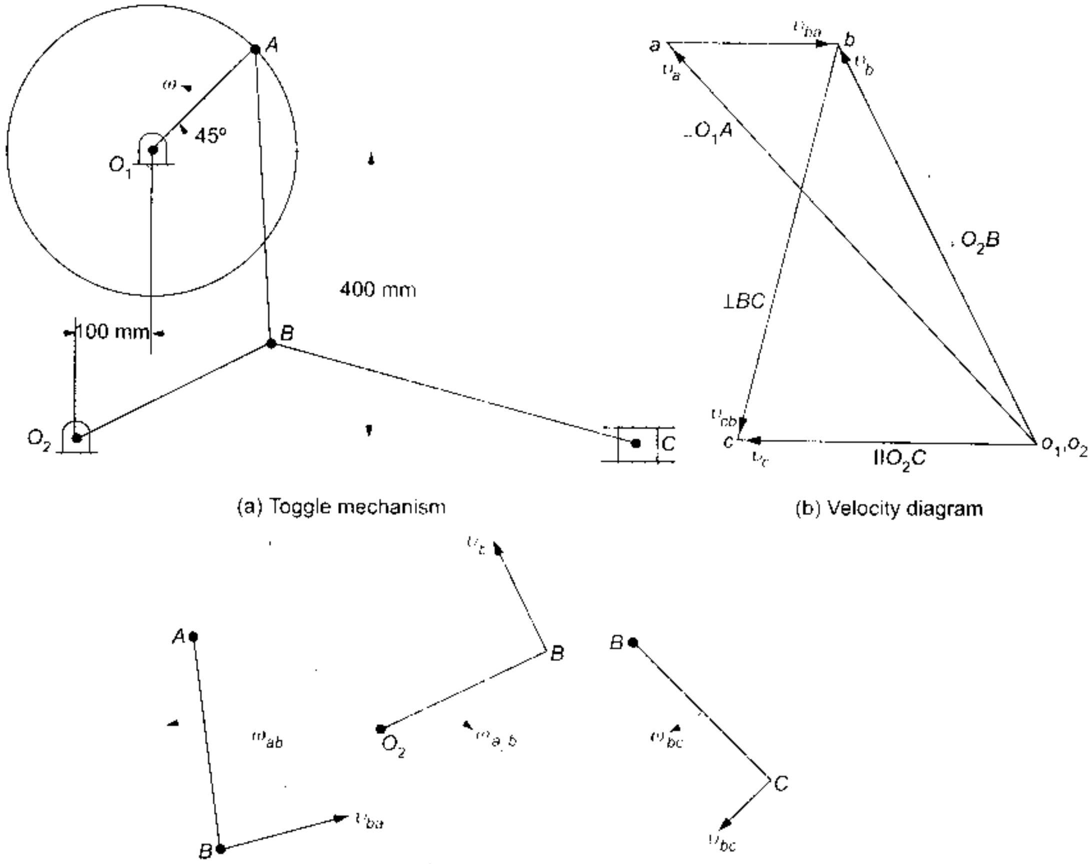
$$v_c = o_1c = 1.4 \text{ cm} = 0.7 \text{ m/s}$$
$$v_{cb} = bc = 1.3 \text{ cm} = 0.65 \text{ m/s}$$

Angular velocity of link $BC = \dfrac{v_{cb}}{BC} = \dfrac{0.65}{0.05} = 13$ rad/s, clockwise about B.

Example 2.2

The slider C of the toggle mechanism shown in Fig.2.15(a), is constrained to move on a horizontal path. The crank O_1A rotates in the counter-clockwise direction at a uniform speed of 180 rpm.

$O_1A = 200$ mm, AB = 400 mm, $O_2B = 300$ mm, and $BC = 600$ mm



(c) Angular velocities

Fig.2.15

Determine *(a)* the velocity of slider c, (b) the angular velocity of links *AB*, O_2B and *BC* (c) the rubbing velocities on the pins of diameter 25 mm at *A* and *C* and (d) the torque required at the crank O_1A for a force of 2 kN at *C*.

■ Solution

$$\omega = 2\pi \times \frac{180}{60} = 18.85 \text{ rad/s}$$

$$v_a = \omega \cdot O_1A = 18.85 \times 0.2 = 2.77 \text{ m/s}$$

(a) Draw $v_a = o_1a \perp O_1A$ to a scale of 1 cm = 0.5 m/s (Fig.2.15b). From *a* draw a line perpendicular to *AB* and another line from o_1 perpendicular to O_2B to intersect at *b*. Then $o_1b = v_b$ and $ab = v_{ba}$. From *b* draw a line perpendicular to *BC* and another line from o_1 parallel to the path of motion of the slider *C*, to meet at *c*. Then $o_1c = v_c$ is the velocity of the slider. By measurement, we get

$$v_c = o_1c = 4.2 \text{ cm} = 2.1 \text{ m/s}$$

(b) $$v_{ba} = ab = 2.4 \text{ cm} = 1.2 \text{ m/s}$$

$$v_b = o_1b = 6.2 \text{ cm} = 2.1 \text{ m/s}$$

$$v_{cb} = bc = 5.7 \text{ cm} = 2.85 \text{ m/s}$$

$$\omega_{ab} = \frac{v_{ba}}{AB} = \frac{1.2}{0.4} = 3 \text{ rad/s (ccw about A)}$$

$$\omega_b = \frac{v_b}{O_2B} = \frac{2.1}{0.3} = 10.33 \text{ rad/s (ccw about } O_2\text{)}$$

$$\omega_{bc} = \frac{v_{cb}}{BC} = \frac{2.85}{0.6} = 4.75 \text{ rad/s (cw about B)}$$

(c) $$r = \frac{25}{2} = 12.5 \text{ mm}$$

$$\text{Rubbing velocity on pin at } C = \omega_{cb} \cdot r = 4.75 \times 0.0125 = 0.0594 \text{ m/s}$$

$$\text{Relative angular velocity at } A = \omega_b - \omega_{ba} + \omega_{cb}$$

$$= 10.33 - 3 + 4.75 = 12.08 \text{ rad/s}$$

$$\text{Rubbing velocity on pin at } A = 12.08 \times 0.0125 = 0.151 \text{ m/s}$$

(d) Let T be the torque required at the crank O_1A.

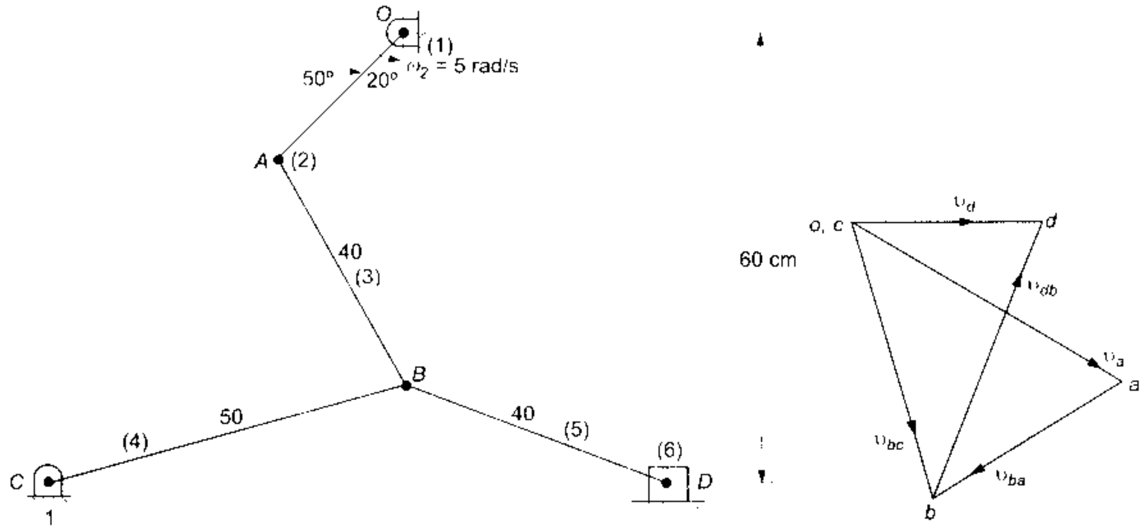
$$T\omega = F_c v_c$$

$$18.85\,T = 2000 \times 2.1$$

$$T = 222.81 \text{ Nm}$$

Example 2.3

Determine the mechanical advantage of the toggle mechanism shown in Fig.2.16(a).



Scale: 1 cm = 10 cm
(a) Configuration diagram

Scale: 1 cm = 20 cm/s
(b) Velocity diagram

Fig.2.16 Toggle mechanism

Solution

$$v_a = \omega_2 \times OA = 5 \times 20 = 100 \text{ cm/s}$$

Draw the velocity diagram to a scale of 1 cm = 20 cm/s, as shown in Fig. (2.16b).

$$v_a = oa \perp OA,\ ab \perp AB,\ cb \perp BC,\ bd \perp BD, \text{ and } cd \parallel CD.$$

Let T_a = torque input at A

$$\text{Force, } F_a = \frac{T_a}{OA}$$

$$\text{Work input} = F_a \times v_a$$

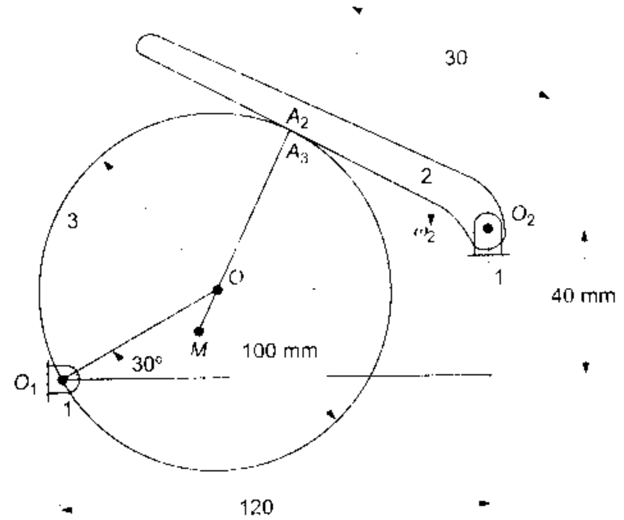
Let F_d = force at D

$$\therefore \text{Work output} = F_d \times v_d$$

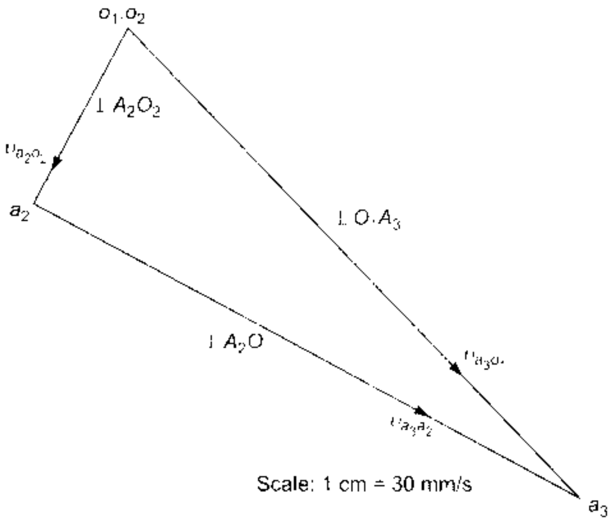
$$\text{Mechanical advantage} = \frac{F_d}{F_a} = \frac{v_a}{v_d} = \frac{oa}{cd} = \frac{5}{2.5} = 2$$

Example 2.4

Determine the angular velocity of the follower 3 and the velocity of sliding at the point of contact in Fig. 2.17(a). The speed of driver link 2 is 3 rad/s.



(a) Configuration diagram



(b) Velocity diagram

Fig.2.17 Mechanism with follower

■ Solution

Join A_2O, O_1O and O_1O_2. Produce A_2O to meet O_1O_2 at M

$$v_{a_2o_2} = \omega_2 \times O_2A_2 = 3 \times 30 = 90 \text{ mm/s}$$

Now $MA_2 \perp O_2A_2$. Therefore,

$$\frac{\omega_3}{\omega_2} = \frac{O_2M}{O_1M}$$

$$\omega_3 = \frac{3 \times 44}{20} = 6.6 \text{ rad/s}$$

Draw the velocity diagram as shown in Fig.2.17(b) to a scale of 1 cm = 30 mm/s.

$$o_1a_2 = v_{a_2o_2} \perp A_2O_2$$
$$o_1a_3 = v_{a_3o} \perp O_1A_3$$
$$a_2a_3 = v_{a_3a_2} \perp a_2O$$

Velocity of sliding, $v_{a_3a_2} = a_2a_3 = 9.4 \text{ cm} = 282 \text{ mm/s}$

Example 2.5

The crank AB of a four-bar mechanism shown in Fig.2.18(a) rotates at 60 rpm clockwise. Determine the relative angular velocities of the coupler to the crank and the lever to the coupler. Find also the rubbing velocities at the surface of pins 25 mm radius at the joints B and C.

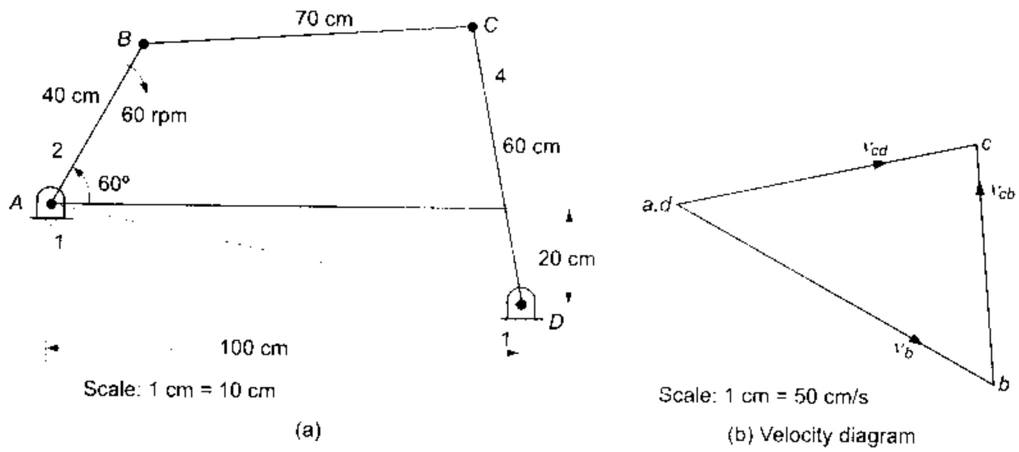


Fig.2.18 Four-bar mechanism

■ Solution

$$\omega_2 = 2\pi \times \frac{60}{60} = 6.28 \text{ rad/s}$$
$$v_b = \omega_2 \times AB = 6.28 \times 40 = 251.3 \text{ cm/s}$$

Draw the velocity diagram as shown in Fig.2.18(b), to a scale of 1 cm = 50 cm/s.

$$v_b = ab \perp AB$$
$$bc \perp BC$$
$$dc \perp CD$$
$$v_{cb} = bc = 2.3 \text{ cm} = 165 \text{ cm/s}$$
$$v_{cd} = dc = 4.2 \text{ cm} = 210 \text{ cm/s}$$
$$\omega_3 = \frac{bc}{BC} = \frac{165}{70} = 2.36 \text{ rad/s (ccw)}$$
$$\omega_{23} = \omega_2 - \omega_3 = 6.28 - (-2.36) = 8.64 \text{ rad/s (cw)}$$

Rubbing velocity of pin $B = \omega_{23} \times r_p = 8.64 \times 2.5 = 21.6$ cm/s

$$\omega_4 = \frac{cd}{CD} = \frac{210}{60} = 2.5 \text{ rad/s (cw)}$$
$$\omega_{34} = \omega_3 - \omega_4 = 2.36 - (-2.5) = 5.86 \text{ rad/s (ccw)}$$

Rubbing velocity of pin $C = \omega_{34} \times r_p = 5.86 \times 2.5 = 14.65$ cm/s.

Example 2.6

Wheel 2 in Fig.2.19(a) rotates at 1500 rpm and is driving the wheel 7 pivoted at O_2. Determine the linear velocity of slider and angular velocities of links 3, 4 and 6.

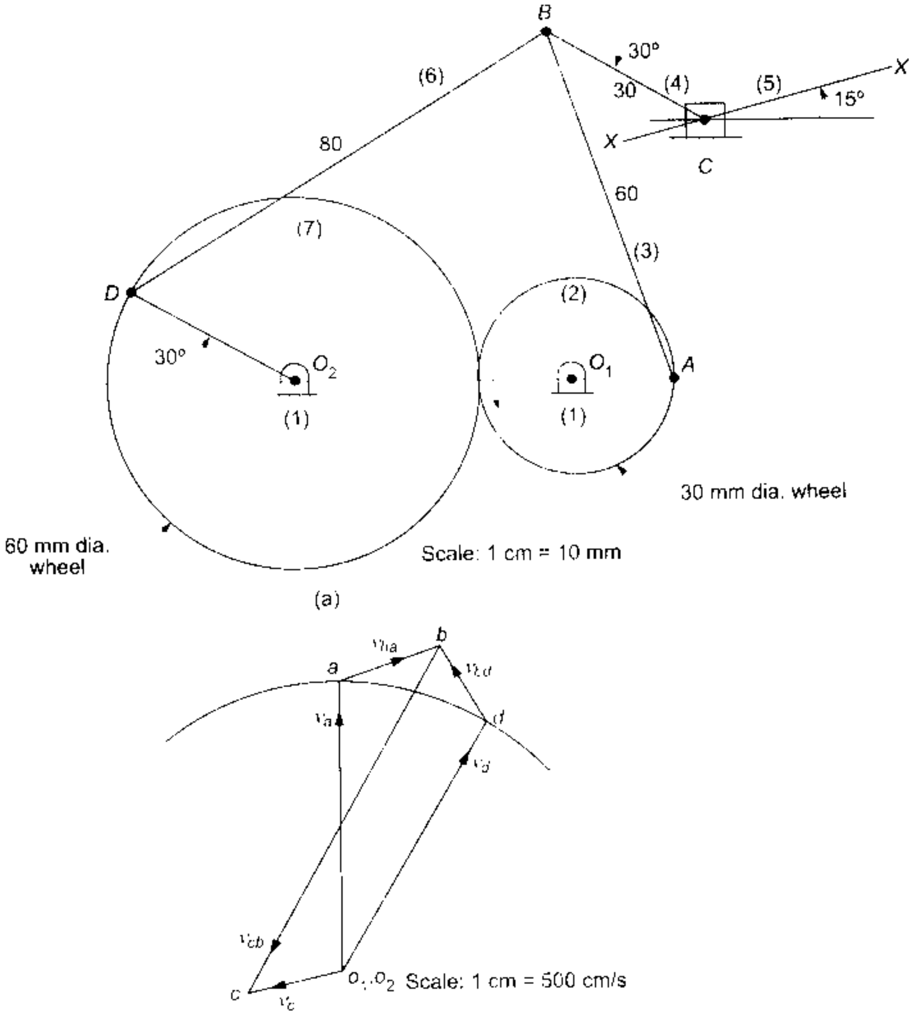


Fig.2.19 Mechanism with wheel

■ **Solution**

$$\omega_2 = 2\pi \times \frac{1500}{60} = 157.08 \text{ rad/s}$$

$$v_a = \omega_2 \times O_1A = 157.08 \times 15 = 2356.2 \text{ mm/s}$$

Draw the velocity diagram as shown in Fig.2.19(b) to a scale of 1 cm = 500 mm/s.

$$v_a = o_1a \perp O_1A \qquad v_d = v_a \perp O_2D$$

$$ab \perp AB \qquad db \perp DB$$

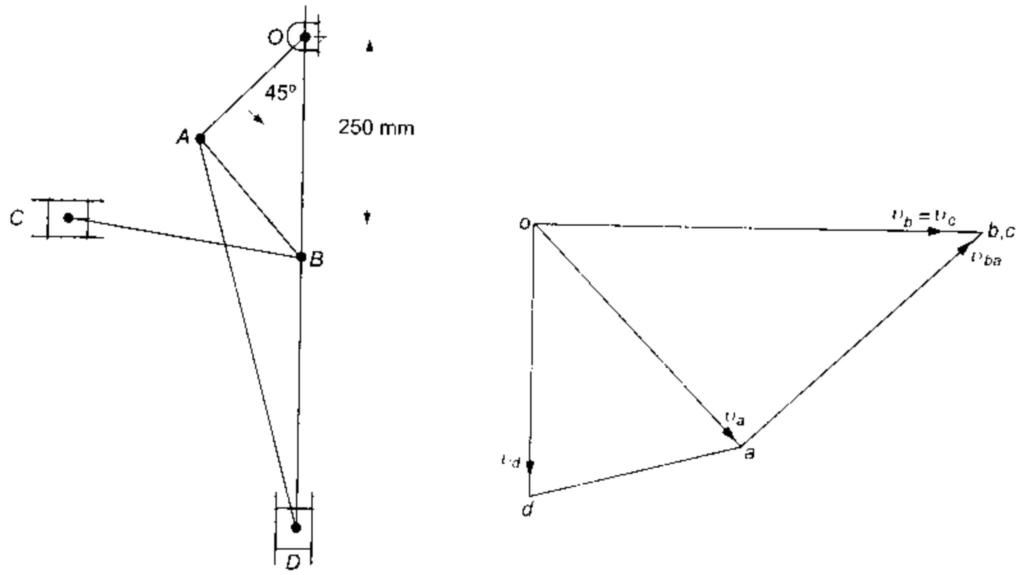
$$bc \perp BC \qquad o_1c \parallel XX$$

Linear velocity of slider $= v_c = o_1c = 1.6$ cm $= 800$ mm/s

$$v_{ba} = ab = 1.7 \text{ cm} = 850 \text{ mm/s}$$
$$\omega_3 = \frac{v_{ba}}{AB} = \frac{850}{60} = 14.16 \text{ rad/s (cw)}$$
$$v_{cb} = bc = 6.5 \text{ cm} = 3250 \text{ mm/s}$$
$$\omega_4 = \frac{v_{cb}}{BC} = \frac{3250}{30} = 108.3 \text{ rad/s (cw)}$$
$$v_{bd} = db = 1.4 \text{ cm} = 700 \text{ mm/s}$$
$$\omega_6 = \frac{v_{bd}}{BD} = \frac{700}{80} = 8.75 \text{ rad/s (ccw)}$$

Example 2.7

The dimensions of the mechanism for hydraulic riveter, as shown in Fig.2.20(a), are: $OA = 200$ mm, $AB = 210$ mm, $AD = 550$ mm and $BC = 330$ mm.



(a) Hydraulic riveter

(b) Velocity diagram

Fig.2.20

Determine the velocity ratio between the piston C and ram D. Also calculate the efficiency of the machine if a load of 3 kN on piston C causes a thrust of 4.5 kN at ram D.

■ Solution

Let N be the speed of the crank OA in rpm. Then $\omega = 2\pi \frac{N}{60}$ rad/s.

$$v_a = \omega \cdot \text{OA} = 2\pi N \times \frac{0.2}{60} = 0.0209 \text{ N m/s}$$

Since N is unknown, let $v_a = 25$ mm. Draw $oa \perp OA$ to represent v_a (Fig.2.20b). From a draw a vector $ab \perp AB$, to represent v_{ba}, and from o draw $ob \perp OB$, to represent v_b, to intersect it at b.

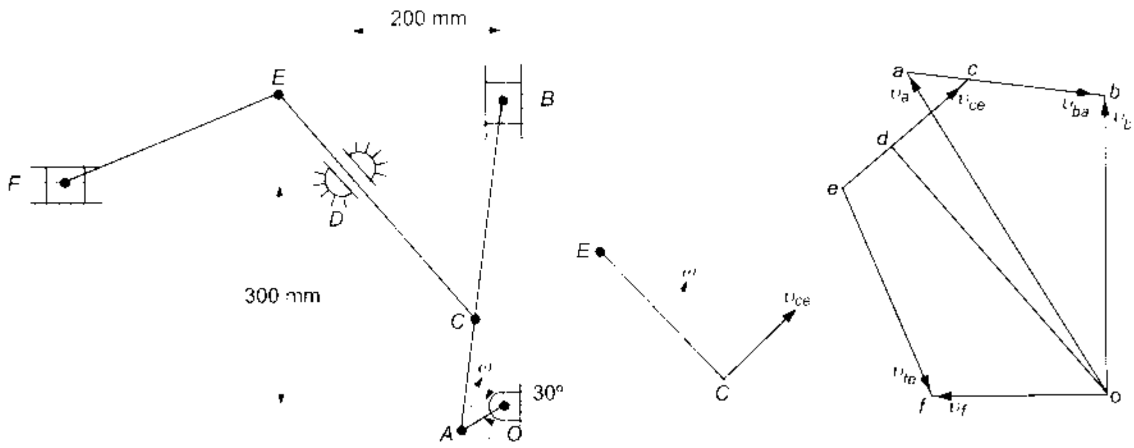
From point b draw a line perpendicular to BC and from o draw a line parallel to the path of motion of the piston C. These lines intersect at c so that b and c coincide. Thus $v_b = v_c$.

From point a draw a line perpendicular to AD and from o draw another line parallel to the path of motion of ram D, to intersect at d. Then $od = v_d$, the velocity of ram D. By measurement, we get

$$v_c = oc = 74 \text{ mm}$$
$$v_d = od = 45 \text{ mm}$$
$$\text{Velocity ratio} = \frac{v_c}{v_d} = \frac{74}{45} = 1.644$$
$$\text{Work done on the piston } C = \text{Load on the piston} \times v_c$$
$$= 3000 v_c$$
$$\text{Work done by ram } D = 4500 v_d$$
$$\text{Efficiency} = \frac{4500 v_d}{3000 v_c}$$
$$= \frac{1.5}{1.644} = 0.912 \quad \text{or} \quad 91.2\%$$

Example 2.8

Crank OA in Fig.2.21(a) is 80 mm long and rotates clockwise about O at 120 rpm. The connecting rod AB is 450 mm long. Point C on AB is such that $AC = 150$ mm. A rod $CE = 400$ mm is attached at point C which slides in a trunnion at D. The end E is connected by a link $EF = 320$ mm to the horizontally moving slider F. Find (a) the velocity of F, (b) the velocity of sliding of CE in the trunnion and (c) the angular velocity of CE.



(a) Configuration diagram (b) Velocity diagram

Fig.2.21

Solution

$$\text{Angular speed of } OA, \quad \omega = 2\pi \times \frac{120}{60} = 12.57 \text{ rad/s}$$
$$v_a = \omega \cdot OA = 12.57 \times 0.08 = 1.005 \text{ m/s}$$

(a) Draw $oa \perp OA$ to represent v_a to a scale of 1 cm = 0.2 m/s, as shown in Fig.2.21(b). From *a* draw a line perpendicular to *AB* and from *o* draw another line parallel to the line of motion of the slider to intersect at *b*. Locate point *c* on *ab* such that, $ac/ab = AC/AB$. From point *c* draw a line perpendicular to *CD* to represent v_{dc} and at point *o* draw another line parallel to the motion of *CD*, which moves along *CD* only, to represent v_d. Locate point *e* on *cd* such that, $cd/ce = CD/CE$. From point *e* draw a line perpendicular to *EF* and from point *o* draw another line parallel to the path of motion of the slider *F*, to meet at point *f*. Then $of = v_f$. By measurement, we have

$$v_f = 2.3 \text{ cm} = 0.46 \text{ m/s}$$

(b) Velocity of sliding of *CE* in trunnion *D* is,

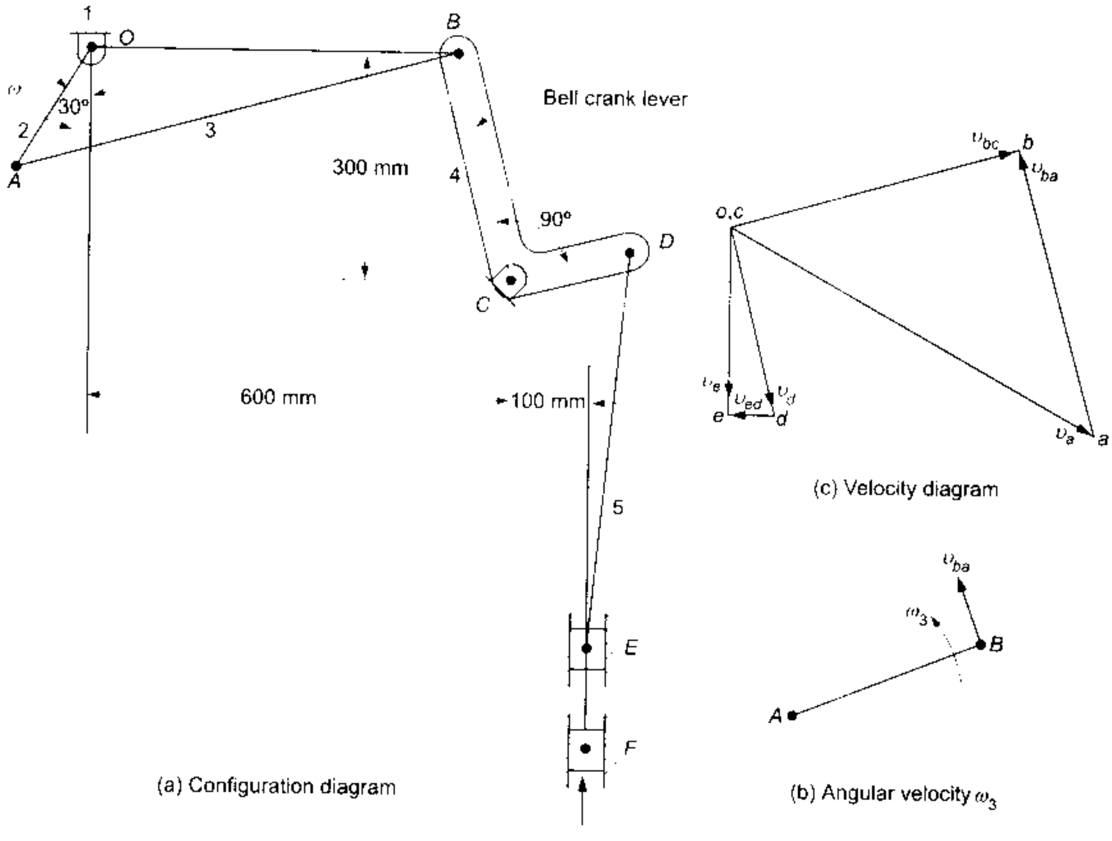
$$v_d = od = 4.3 \text{ cm} = 0.86 \text{ m/s}$$

(c) $v_{ce} = ec = 2.4 \text{ cm} = 0.48 \text{ m/s}$

Angular velocity of $CE = \frac{v_{ce}}{CE} = \frac{0.48}{0.4} = 1.2$ rad/s ccw about *E*.

Example 2.9

An engine crankshaft drives a reciprocating pump through a mechanism, as shown in Fig.2.22(a). The crank *OA* rotates in the counter-clockwise direction at 150 rpm. The diameter of the pump piston at *F* is 180 mm and *OA* = 175 mm, *AB* = 650 mm, *CD* = 160 mm, and *DE* = 600 mm.



(a) Configuration diagram

Fig.2.22

Determine (a) the velocity of cross head E, (b) the rubbing velocities at pins A, B, C, and D having diameters of 40 mm each and (c) the torque required at the crank to overcome a pressure of 0.35 MPa at the pump piston at F.

■ Solution

$$\omega = 2\pi \times \frac{150}{60} = 15.708 \text{ rad/s}$$

$$v_a = 15.708 \times 0.175 = 2.75 \text{ m/s}$$

Draw $oa \perp OA$ to represent v_a to a scale of 1 cm = 0.5 m/s, as shown in Fig.2.22(b). From a draw a line perpendicular to AB and from c draw another line perpendicular to BC to intersect at B. Then $ab = v_{ba}$ and $cb = v_{bc}$.

$$cd = cb \cdot \left(\frac{CD}{BC}\right) = 5 \times \frac{160}{310} = 2.58 \text{ cm}$$

Draw a line perpendicular to CD from c and cut it equal to cd. From d draw a line perpendicular to DE and from o draw another line parallel to the path of motion of the slider E, to meet at e. Then $de = v_{ed}$ and $oe = v_e$. By measurements, we have

(a) Velocity of cross-head E,

$$v_e = oe = 2.5 \text{ cm} = 1.25 \text{ m/s}$$

(b) Rubbing velocities,

$$\omega_2 = 15.708 \text{ rad/s ccw about } O$$

$$\omega_3 = \frac{ab}{AB} = \frac{v_{ba}}{AB} = 2.9 \times \frac{0.5}{0.650} = 3 \text{ rad/s ccw about } A$$

$$\omega_{32} = \omega_3 - \omega_2 = 3 - 15.708 = -12.708 \text{ rad/s}$$

Pin A:

$$\omega_{32} \times 0.020 = -0.254 \text{ m/s}$$

$$\omega_3 = \frac{ba}{AB} \text{ ccw about } B = 2.9 \times \frac{0.5}{0.65} = 3 \text{ rad/s}$$

$$\omega_4 = \frac{bc}{BC} \text{ cw about } B = 4 \times \frac{0.5}{0.31} = 6.45 \text{ rad/s}$$

$$\omega_{43} = \omega_4 - \omega_3 = 6.45 - 3 = 9.45 \text{ rad/s}$$

Pin B:

$$\omega_{43} \times 0.020 = 9.45 \times 0.020 = 0.189 \text{ m/s}$$

Pin C:

$$\left(\frac{cb}{BC}\right) \times 0.020 = (4 \times 0.5/0.31)\,0.020 = 0.129 \text{ m/s}$$

$$\omega_5 = \frac{de}{DE} \text{ cw about } D$$

$$= 0.6 \times \frac{0.5}{0.6} = 0.5 \text{ rad/s}$$

$$\omega_4 = \frac{dc}{DC} \text{ cw about } D$$

$$= 2.6 \times \frac{0.5}{0.160} = 8.125 \text{ m/s}$$

$$\omega_{54} = \omega_5 - \omega_4 = 0.5 - 8.125 = -7.625 \text{ rad/s}$$

Pin D:

$$\omega_{54} \times 0.020 = 7.625 \times 0.020 = 0.1525 \text{ m/s}$$

(c) Velocity of the piston, $v_f = v_e = 2.5 \times 0.5 = 1.25$ m/s

Let T be the torque required at the crank OA. Then,

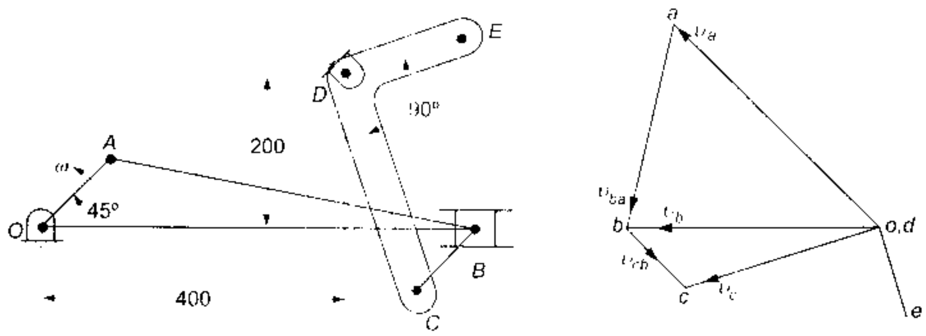
$$T\omega = F v_f$$

$$15.708T = \left(\frac{\pi}{4}\right)(180)^2 \times 0.35 \times 10^6 \times 1.25$$

$$T = 708.75 \text{ Nm}$$

Example 2.10

The various dimensions of the mechanism, as shown in Fig.2.23(a), are OA = 120 mm, AB = 500 mm, BC = 120 mm, CD = 300 mm and DE = 150 mm. The crank OA rotates at 150 rpm. The bell crank lever is DE. Determine the absolute velocity of point E.



(a) Configuration diagram (b) Velocity diagram

Fig.2.23

■ **Solution**

$$\omega = 2\pi \times \frac{150}{60} = 15.708 \text{ rad/s}$$

$$v_a = 15.708 \times 0.120 = 1.885 \text{ m/s}$$

Draw $oa \perp OA$ to represent the velocity v_a of point A to a scale of 1 cm = 0.5 m/s, as shown in Fig. 2.23(b). At a draw a line perpendicular to AB and at o draw another line parallel to the path of motion of the slider at B to intersect at b. Then $ob = v_b$ and $ab = v_{ba}$.

At b draw a line perpendicular to BC and at d draw another line perpendicular to DC to intersect at c. Then $bc = v_{cb}$ and $dc = v_c$. Now draw $de \perp dc$ such that

$$\frac{de}{dc} = \frac{DE}{DC}$$

or

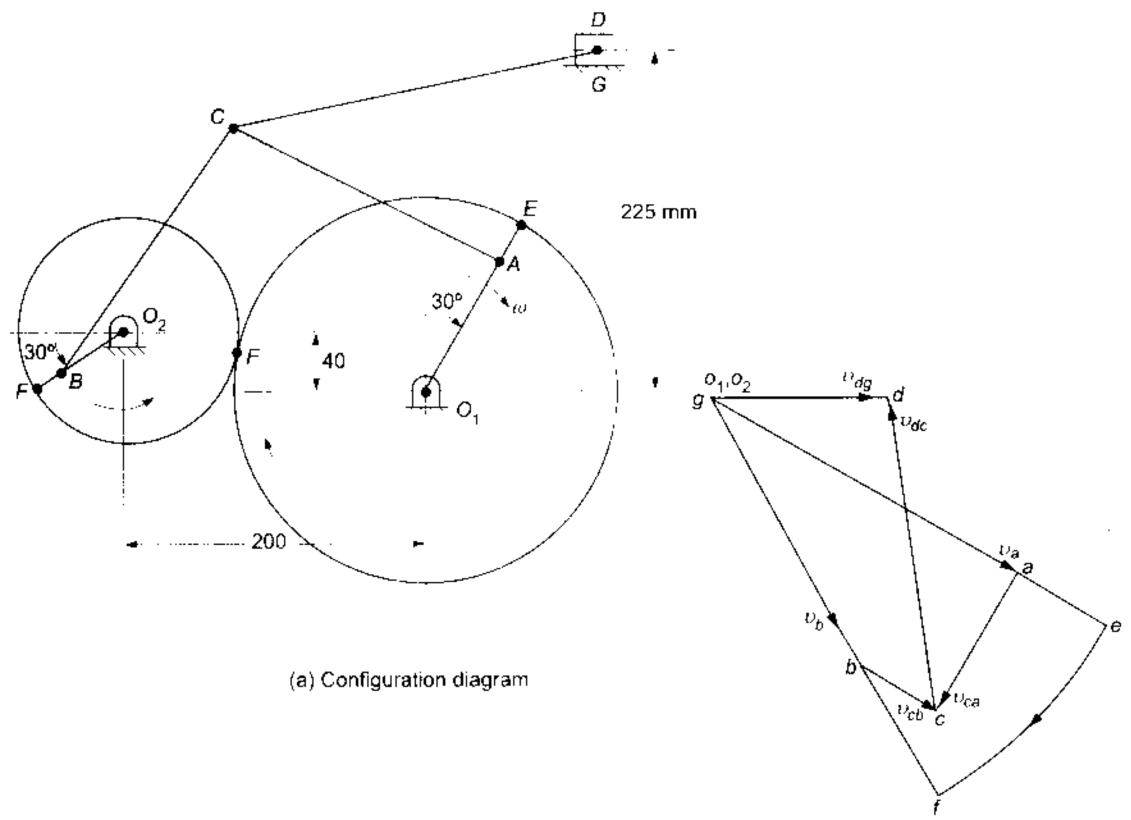
$$de = 2.5 \times \frac{150}{300} = 1.25 \text{ cm}$$

Then

$$v_e = de = 1.25 \text{ cm} = 0.625 \text{ m/s}$$

Example 2.11

In the mechanism shown in Fig.2.24(a), the crank O_1A and O_2B are 100 mm and 50 mm, respectively. The diameters of wheels with centres O_1 and O_2 are 260 mm and 150 mm, respectively. $BC = AC$ = 200 mm, CD = 250 mm. The wheels roll on each other. The crank O_1A rotates at 120 rpm. Determine (a) the velocity of the slider D, (b) the angular velocities of links BC and CD and (c) the torque at O_2B when the force required at D is 4 kN.



(a) Configuration diagram

(b) Velocity diagram

Fig.2.24

■ **Solution**

$$\omega = 2\pi \times \frac{120}{60} = 12.57 \text{ rad/s}$$

$$v_a = 12.57 \times 0.1 = 1.257 \text{ m/s}$$

$$v_e = v_a \times \frac{O_1E}{O_1A} = 1.257 \times \frac{130}{100} = 1.634 \text{ m/s}$$

$$v_f = v_e$$

$$v_b = v_f \times \frac{O_2B}{O_2F} = 1.634 \times \frac{50}{60} = 1.362 \text{ m/s}$$

Draw $o_1a \perp O_1A$ to represent the velocity v_a of point A to a scale of 1 cm = 0.25 m/s, as shown in Fig.2.24(b). Extend o_1a to e such that

$$o_1e = o_1a \times \frac{O_1E}{O_1A}$$

$$= 5 \times \frac{130}{100} = 6.5 \text{ cm}$$

From O_2 draw a line perpendicular to O_2B to intersect o_1e rotated about o_1 at f. Locate point b on o_2f such that

$$o_2b = o_2f \times \frac{O_2B}{O_2F}$$

$$= 6.5 \times \frac{50}{75} = 4.33 \text{ cm}$$

Draw a line perpendicular to *BC* at *b* and draw another line perpendicular to *AC* at *a* to intersect at *c*. Now draw a line perpendicular to *CD* at *c* and draw another line from *g* parallel to the path of motion of slider *D* to intersect at *d*. Then,

(a) the velocity of slider, $v_{dg} = gd = 2.4 \text{ cm} = 0.6$ m/s

(b) $v_{bc} = cb = 1.4 \text{ cm} = 0.35$ m/s

$$\omega_{bc} = \frac{v_{bc}}{BC} = \frac{0.35}{0.2} = 1.75 \text{ rad/s (cw)}$$

$$v_{dc} = cd = 4.6 \text{ cm} = 1.15 \text{ m/s}$$

$$\omega_{cd} = \frac{v_{dc}}{DC} = \frac{1.15}{0.25} = 4.6 \text{ rad/s (ccw)}$$

(c) $T\omega = F_d\, v_{dg}$

$$T = 4000 \times \frac{0.6}{12.57} = 190.93 \text{ Nm}$$

Example 2.12

In the mechanism shown in Fig.2.25(a), $v_a = 120$ m/s. Determine the angular velocities ω_4, ω_5 of the two gears and the velocity v_d on gear 5. $O_2A = 50$ mm, $AB = 200$ mm and $O_6C = 150$ mm.

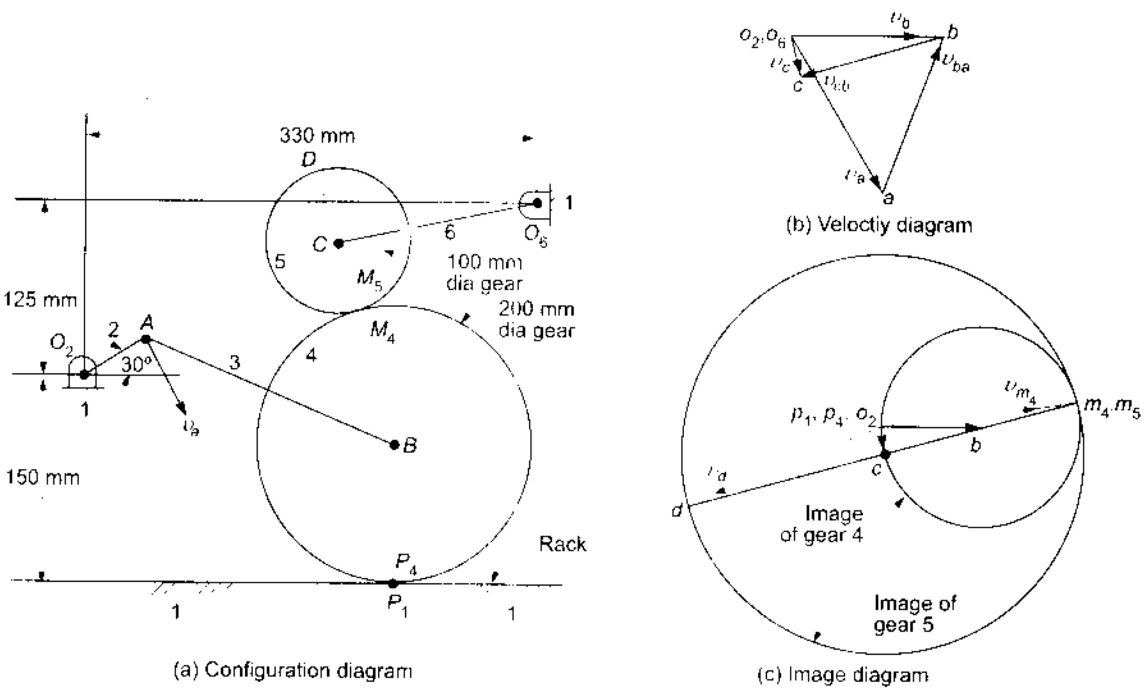


Fig.2.25

■ **Solution**

Draw $o_2a \perp O_2A$ to represent $v_a = 120$ m/s to a scale of 1 cm = 40 m/s, as shown in Fig.2.25(b). Draw $ab \perp AB$ and o_2b parallel to pitch line of rack. Then $o_2b = v_b$ and $ab = v_{ba}$. Draw $bc \perp BC$ and $o_6c \perp O_6C$. Then $o_6c = v_c$ and $bc = v_{cb}$. By measurement,

$$v_b = o_2b = 2.6 \text{ cm} = 104 \text{ m/s}$$

$$
\begin{aligned}
v_c &= o_6c = 0.7 \text{ cm} = 28 \text{ m/s} \\
v_{ba} &= ab = 2.9 \text{ cm} = 116 \text{ m/s} \\
v_{cb} &= bc = 2.6 \text{ cm} = 104 \text{ m/s} \\
\omega_4 &= \frac{(v_{bp4} = v_b)}{BP} = \frac{104}{0.1} = 1040 \text{ rad/s (cw)} \\
\omega_5 &= \frac{v_{cm5}}{CM} = 5.2 \times \frac{40}{0.05} = 4160 \text{ rad/s (ccw)} \\
v_{m4} &= v_{m5} = 5.2 \text{ cm} = 208 \text{ m/s} \\
v_d &= 5.3 \text{ cm} = 212 \text{ m/s}
\end{aligned}
$$

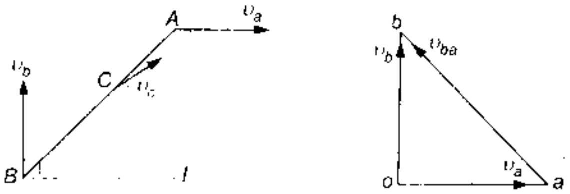
Because $v_{p1} = 0$ and $v_{p4} = v_{p1}$, the images of the points P_1 and P_4 both are at the pole point o_2. Draw velocity image of gear 4 with b as centre and radius bp_4. Produce cb to meet the circle at m_4, m_5 since $v_{m4} = v_{m5}$. The velocity image of gear 5 is drawn with c as centre and cm_5 as radius. Point d is located on the circle opposite to m_5.

2.4 INSTANTANEOUS CENTRE METHOD

A link as a whole may be considered to be rotating about an imaginary centre or about a given centre at a given instant. Such a centre has zero velocity, the link is at rest at this point. This is known as the instantaneous centre or centre of rotation. This centre varies from instant to instant for different positions of the link. The locus of these centres is termed the *centrode*.

2.4.1 Velocity of a Point on a Link

Consider two points A and B on a rigid link, having velocities v_a and v_b respectively, as shown in Fig. 2.26(a). From A and B draw lines perpendicular to the directions of motion and let them meet at I. Then I is the instantaneous centre of rotation of the link AB for its given position.



(a) Instantaneous centre (b) Velocity diagram

Fig.2.26

If ω is the instantaneous angular velocity of the link AB, then $v_a = \omega \cdot IA$ and $v_b = \omega \cdot IB$. Thus

$$v_b = \left(\frac{IB}{IA}\right) \cdot v_a \tag{2.12}$$

The velocity diagram for the link AB has been drawn in Fig.2.26(b). Triangles IAB and oab are similar. Hence

$$\frac{oa}{IA} = \frac{ob}{IB} = \frac{ab}{AB}$$

or

$$\frac{v_a}{IA} = \frac{v_I}{IB} = \frac{v_{ba}}{AB} = \frac{v_c}{IC} = \omega \tag{2.13}$$

where C is any point on the link AB.

2.4.2 Properties of Instantaneous Centre

The properties of the instantaneous centre are as follows:

1. At the instantaneous centre of rotation, one rigid link rotates instantaneously relative to another, for the configuration of the mechanism considered.
2. The two rigid links have no linear velocities relative to each other at the instantaneous centre.
3. The two rigid links have the same linear velocity relative to the third rigid link, or any other link.

2.4.3 Number of Instantaneous Centres

The number of instantaneous centres in a mechanism is equal to the number of possible combinations of two links. The number of instantaneous centres,

$$N = \frac{n(n-1)}{2} \tag{2.14}$$

where n = number of links.

2.4.4 Types of Instantaneous Centres

The instantaneous centres for a mechanism are of the following types:

1. Fixed instantaneous centres.
2. Permanent instantaneous centres.
3. Neither fixed nor permanent instantaneous centres.

Consider a four-bar mechanism shown in Fig.2.27. For this mechanism, $n = 4$. Hence $N = 4(4-1)/2 = 6$.

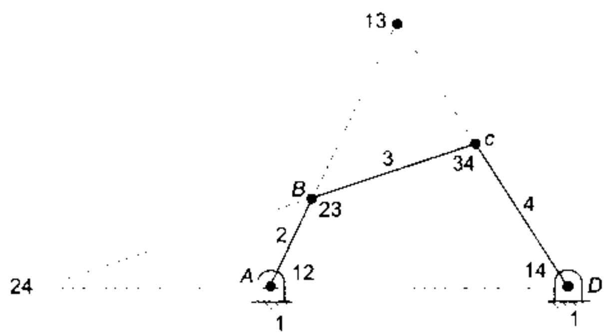


Fig.2.27 Instantaneous centres in a four-bar mechanism

The instantaneous centres are: 12, 13, 14, 23, 24, 34

The instantaneous centres 12 and 14 remain at the same place for the configuration of the mechanism and are therefore called fixed instantaneous centres. The instantaneous centres 23 and 34 move when the mechanism moves, but the joints are permanent. Therefore, they are called permanent instantaneous centres. The instantaneous centres 13 and 24 vary with the configuration of the mechanism and are neither fixed nor permanent.

2.4.5 Location of Instantaneous Centres

The following observations are quite helpful in locating the instantaneous centres:

1. For a pivoted or pin joint, the instantaneous centre for the two links lies on the centre of the pin (see Fig. 2.28a).

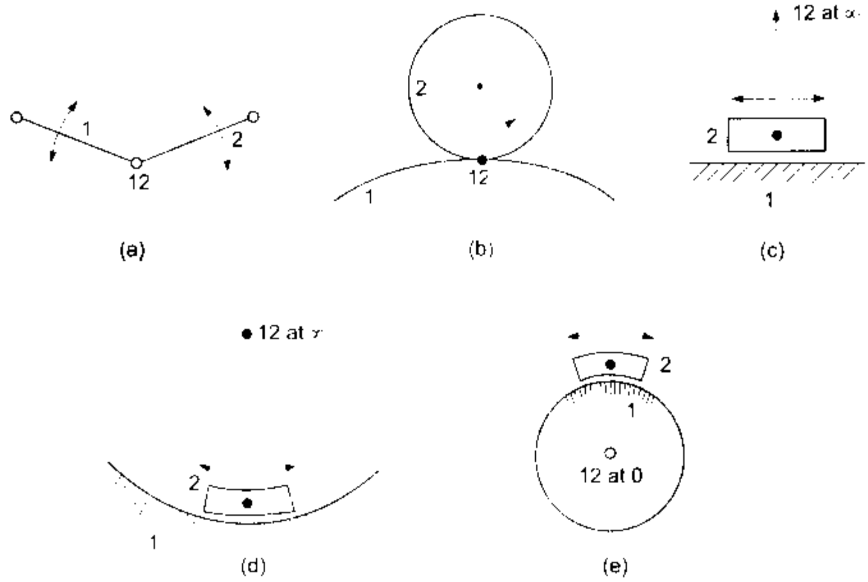


Fig.2.28 Location of instaneous centres

2. When the two links are in pure rolling contact, the instantaneous centre lies at their point of contact (see Fig.2.28b). This is because the relative velocity between the two links at the point of contact is zero.

3. In sliding motion, the instantaneous centre lies at infinity in a direction perpendicular to the path of motion of the slider. This is because the sliding motion is equivalent to a rotary motion of the links with radius of curvature equal to infinity (see Fig.2.28c). If the slider (link 2) moves on a curved surface (link 1), then the instantaneous centre lies at the centre of curvature of the curved surface (see Fig.2.28d and e).

2.4.6 Arnold–Kennedy Theorem

This theorem states that if three plane bodies are in relative motion, their three instantaneous centres must lie on a straight line.

Consider three rigid links 1, 2 and 3; 1 being a fixed link. 12 and 13 are the instantaneous centres of links 1, 2 and 1, 3 respectively. Let 23 be the instantaneous centre of links 2, 3, lying outside the line joining 12 and 13, as shown in Fig.2.29. The links 2 and 3 are moving relative to link 1. Therefore, the motion of their instantaneous centre 23 is to be the same whether it is considered in body 2 or 3. If the point 23 is considered on link 2, then its velocity v_2 is perpendicular to the line joining 12 and 22. If the point 23 lies on link 3, then its velocity v_3 must be perpendicular to the line joining 13 and 22. The velocities v_2 and v_3 of point 23 are in different directions, which is not possible. The velocities v_2 and v_3 of instantaneous centre 23 will be equal only if it lies on the line joining 12 and 13. Hence all the three instantaneous centres 12, 13 and 23 must lie on a straight line.

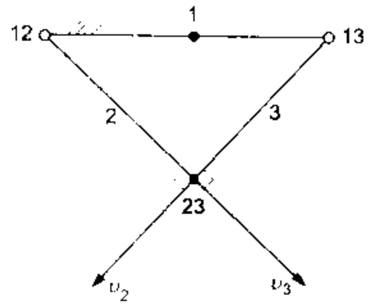


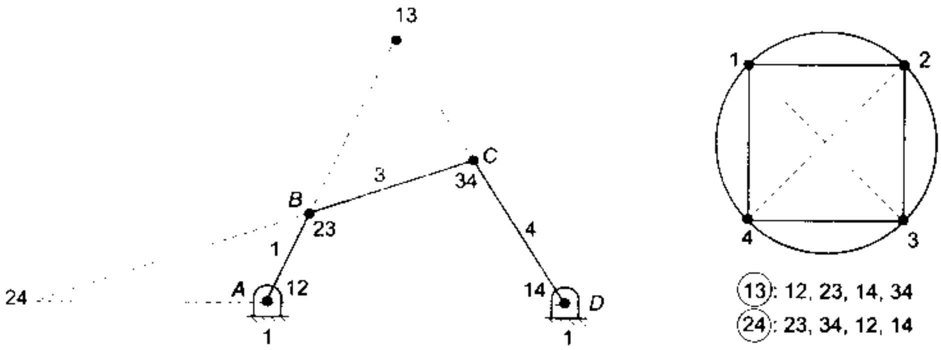
Fig.2.29 Arnold-Kennedy theorem

2.4.7 Method of Locating Instantaneous Centres

The following procedure may be adopted to locate the instantaneous centres:

1. Determine the number of instantaneous centres from $N = \dfrac{n(n-1)}{2}$.
2. Make a list of all the instantaneous centres by writing the link numbers in the first row and instantaneous centres in ascending order in columns. For example, for a four-bar chain, shown in Fig.2.30(a), we have

1	2	3	4
√	√	√	
12	23	34	
(13)	(24)		
√			
14			



(a) Four-bar mechanism (b) Circle diagram

Fig.2.30 Locating instantaneous centres of a four-bar mechanism

3. Locate the fixed and permanent instantaneous centres by inspection, as explained in Section 2.4.5. Tick mark (√) these instantaneous centres.

4. Locate the remaining neither fixed nor permanent instantaneous centres (circled) by using Arnold-Kennedy's theorem. This is done by a circle diagram, as shown in Fig.2.30(b). Mark points on a circle equal to the number of links in the mechanism. Join the points by solid lines for which instantaneous centres are known by inspection. Now join the points forming the other instantaneous centres by dotted lines. The instantaneous centre shall lie at the intersection of the lines joining the instantaneous centres of the two adjacent triangles of the dotted line. For example, in Fig.2.30(b), the centre 13 is located at the intersection of lines (produced) joining the instantaneous centres 12, 23 and 14, 34. Similarly, the centre 24 is located at the intersection of the lines (produced) joining the centres 23, 34 and 12, 14.

2.4.8 Determination of Angular Velocity of a Link

The angular velocities of two links vary inversely as the distances from their common instantaneous centre to their respective centres of rotation relative to the frame. For example, for the four-bar mechanism shown in Fig.2.30(a), if ω_2 is the angular velocity of link 2, then angular velocity ω_4 of link 4 will be given by the following relationship:

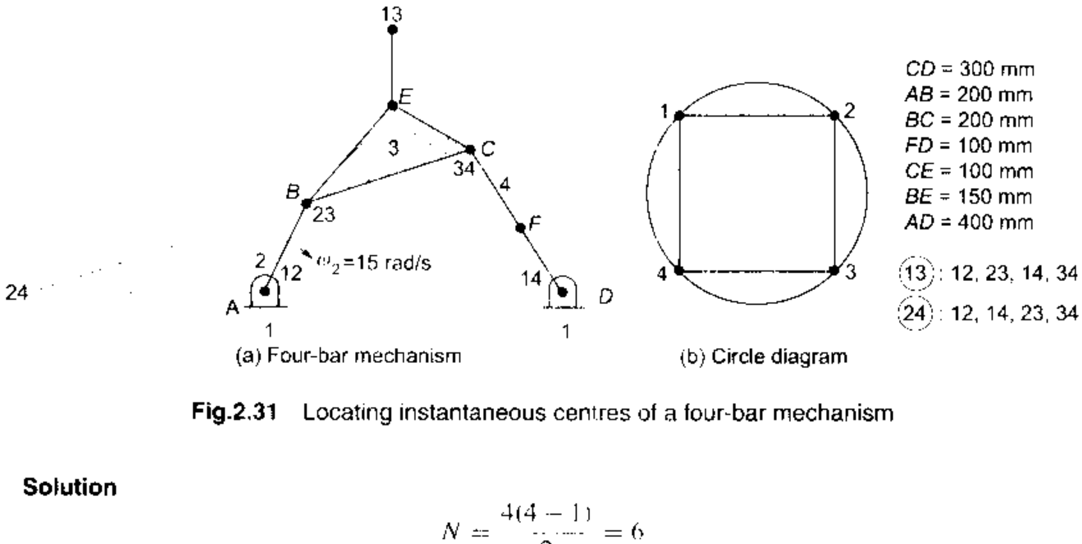
$$\frac{\omega_4}{\omega_2} = \frac{(24 - 12)}{(24 - 14)}$$

If the respective centres of rotation are on the same side of the common instantaneous centre, then the direction of angular velocities will be same. However, if the respective centres of rotation are on opposite sides, then the direction of angular velocities will be opposite.

Similarly,
$$\frac{\omega_3}{\omega_2} = \frac{(23 - 12)}{(23 - 13)}$$

Example 2.13

In the four-bar mechanism shown in Fig.2.31(a), link 2 is rotating at angular velocity ω_2. Locate all the instantaneous centres of the mechanism and find (a) the angular speeds of links 3 and 4, (b) the linear velocities of links 3 and 4 and (c) the linear velocities of points *E* and *F*.



(a) Four-bar mechanism (b) Circle diagram

Fig.2.31 Locating instantaneous centres of a four-bar mechanism

■ **Solution**

$$N = \frac{4(4-1)}{2} = 6$$

The instantaneous centres are:

1	2	3	4
√	√	√	
12	23	34	
(13)	(24)		
14			

As shown in Fig.2.31(b), we have

(13) : 12, 23; 14, 34

(24) : 12, 14; 23, 34

(a)
$$\frac{\omega_3}{\omega_2} = \frac{(23-12)}{(23-13)} = \frac{200}{250}$$

or
$$\omega_3 = 0.8 \times 15 = 12 \text{ rad/s}$$

$$\frac{\omega_4}{\omega_2} = \frac{(24-12)}{(24-14)} = \frac{380}{780}$$

or
$$\omega_4 = \frac{380 \times 15}{780} = 7.31 \text{ rad/s}$$

(b)
$$v_b = \omega_2 \cdot AB = 15 \times 0.2 = 3 \text{ m/s}$$

$$v_c = v_b \left[\frac{13-34}{13-23}\right] = 3 \times \frac{200}{250} = 2.4 \text{ m/s}$$

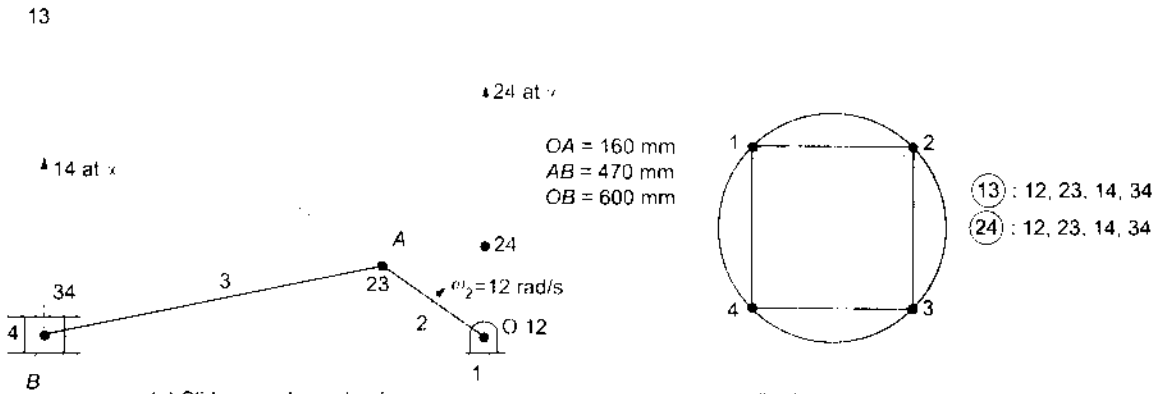
$$v_{bc} = v_b \left[\frac{BC}{13-23}\right] = 3 \times \frac{200}{250} = 2.4 \text{ m/s}$$

(c)
$$v_e = \omega_3 \cdot (13-E) = 12 \times 0.12 = 1.44 \text{ m/s}$$

$$v_f = v_c \left(\frac{FD}{CD}\right) = 2.4 \times \frac{100}{300} = 0.8 \text{ m/s}$$

Example 2.14

Locate the instantaneous centres of the slider-crank mechanism shown in Fig.2.32(a). Find the velocity of the slider.



(a) Slider–crank mechanism (b) Circle method

Fig.2.32 Locating instaneous centres of the slider–crank mechanism

■ **Solution**

$$N = \frac{4(4-1)}{2} = 6$$

1	2	3	4
√	√	√	
12	23	34	
(13)	(24)		
14			

The instantaneous centre 14 is at infinity. As shown in Fig.2.32(b), we have

13 : 12, 23; 14, 34

24 : 12, 14; 23, 34

$$v_a = \omega_2 \cdot OA = 12 \times 0.16 = 1.92 \text{ m/s} = \omega_3 \cdot (23 - 13)$$

Hence,
$$\omega_3 = \omega_2 \left[\frac{23-12}{23-13}\right] = 12 \times \frac{160}{550} = 2.491 \text{ m/s}$$

where ω_3 is the angular velocity of link 3 about 12.

Velocity of the slider,
$$v_b = \omega_3(34-13) = \omega_2 \left[\frac{23-12}{23-13}\right](34-13)$$
$$= \omega_2 \cdot OA \left[\frac{34-13}{23-13}\right] = 12 \times 0.16 \times \frac{390}{550} = 1.36 \text{ m/s.}$$

Example 2.15

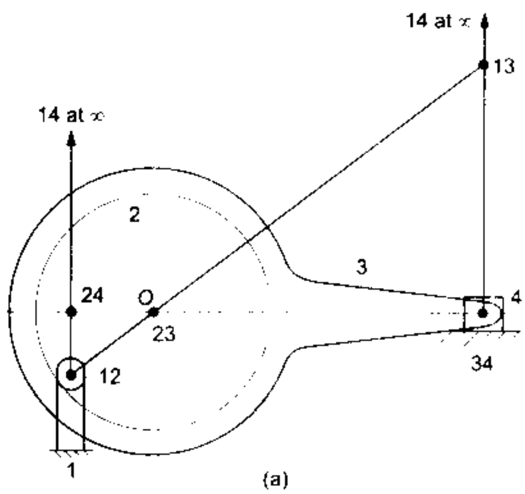
Locate the instantaneous centres of the mechanisms shown in Fig.2.33(a) and (b).

■ **Solution**

(a) Here $n = 4$

Number of instantaneous centres $= \dfrac{4 \times 3}{2} = 6$

1	2	3	4
√	√	√	
12	23	34	
13	24		
√			
14			



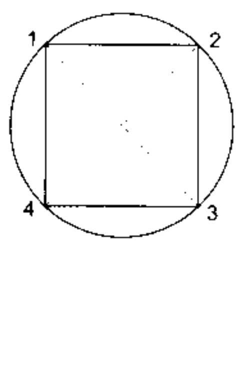


Fig.2.33 Locating instantaneous centres of mechanisms

Instantaneous centres 12, 14, 23, 34 are located by inspection. Instantaneous centres 13 and 24 are located as follows:

(13) : 12, 23; 14, 34

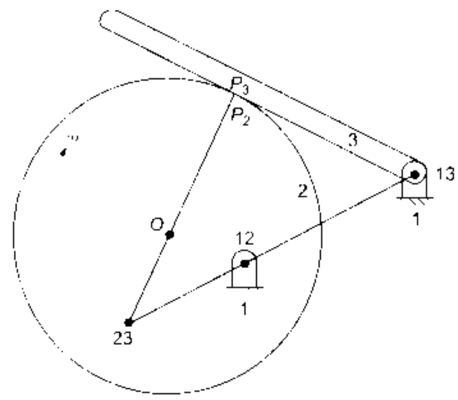
(24) : 23, 34; 12, 14

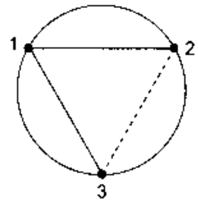
1 2 3
√
12 (23)
√
13

(b) Here $n = 3$; Number of instantaneous centres $= \frac{3 \times 2}{2} = 3$

Instantaneous centres 12, 13 are located by inspection.

(23) : 12, 13





Example 2.16

In the toggle mechanism, shown in Fig.2.34, crank O_1A rotates at 30 rpm clockwise. $O_1A = 40$ mm, $AB = 140$ mm, $BC = 100$ mm, $BD = 80$ mm and $DE = 80$ mm. Neglecting friction and inertia effects, calculate the torque required to overcome a resistance of 500 N at D. Use the instantaneous centre method.

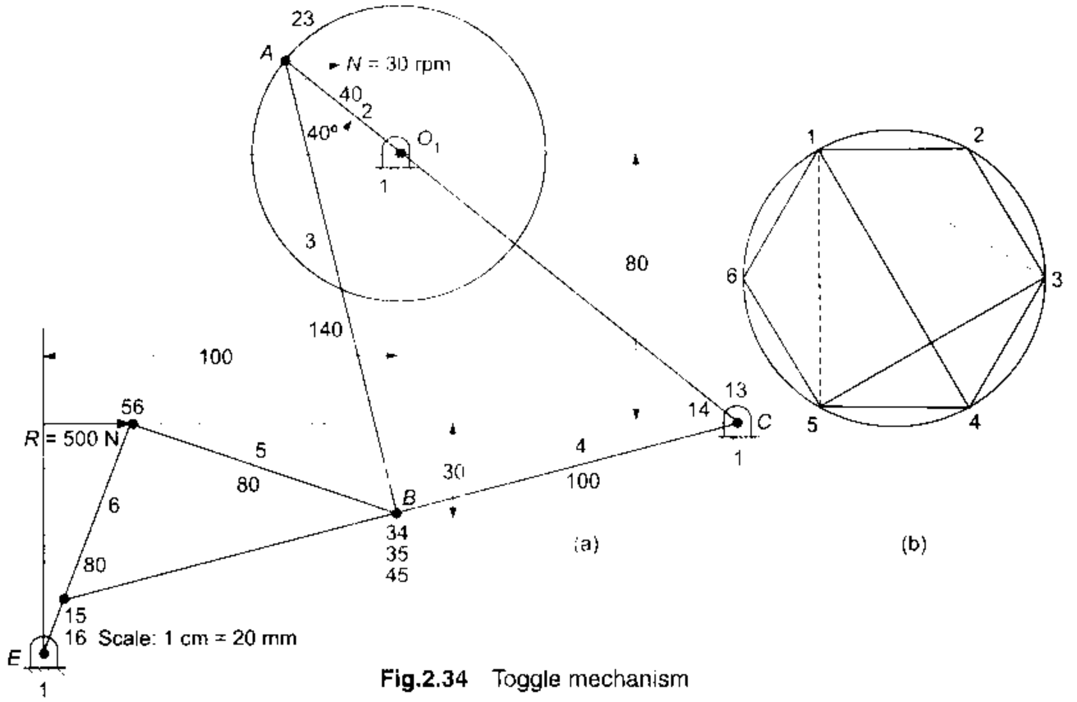


Fig.2.34 Toggle mechanism

■ Solution

Here $n = 6$; Number of instantaneous centres $= \dfrac{n(n-1)}{2} = \dfrac{6 \times 5}{2} = 15$

1	2	3	4	5	6
√	√	√	√	√	
12	23	34	45	56	
		√	√		
(13)	(24)	35	46		
√	√	√			
14	25	36			
(15)	26				
√					
16					

13 : 12, 23; 14, 34
15 : 16, 56; 14, 45
24 : 23, 34; 12, 14

$$\omega_2 = 2\pi \times \frac{30}{60} = 2.14 \text{ rad/s}; \qquad v_a = \omega_2 \times O_1A = 2.14 \times 40 = 125.7 \text{ mm/s}$$

$$\omega_3 = \frac{v_a}{(13-23)} = \frac{125.7}{(8.4 \times 20)} = 0.748 \text{ rad/s}; \qquad v_b = \omega_3(13-34) = 0.748 \times 5.3 \times 20 = 79.29 \text{ mm/s}$$

$$\omega_5 = \frac{v_b}{(15-35)} = \frac{79.29}{(4.8 \times 20)} = 0.826 \text{ rad/s}; \qquad v_d = \omega_5(15-56) = 0.826 \times 2.1 \times 20 = 51.21 \text{ mm/s}$$

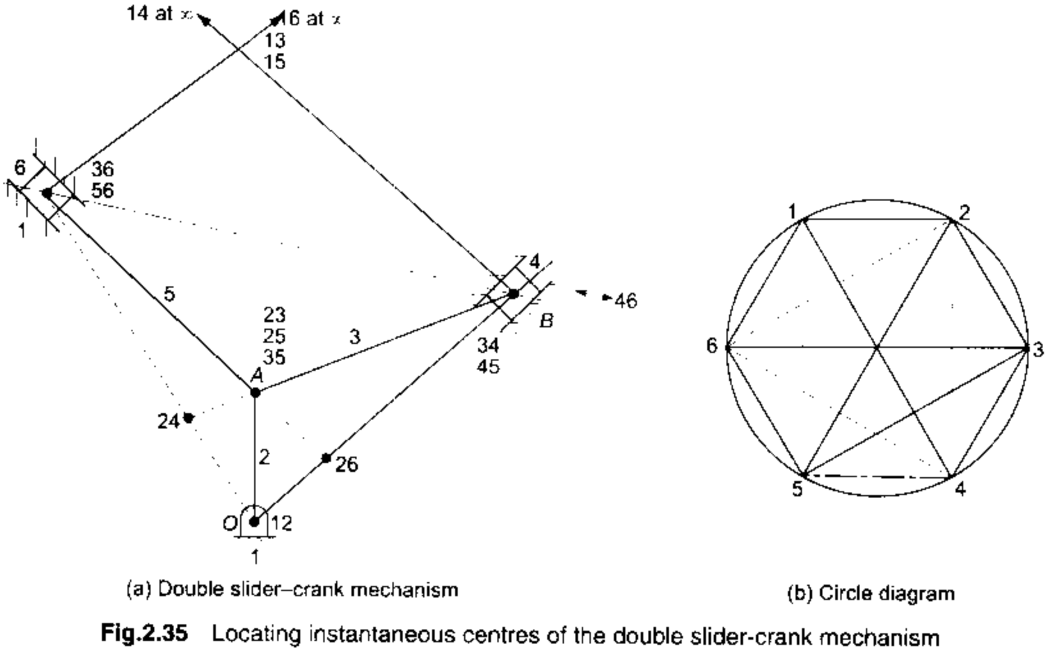
$$\omega_6 = \frac{v_d}{(16-56)} = \frac{51.21}{80} = 0.64 \text{ rad/s}$$

$$\text{Torque} \times \text{Angular velocity} = \text{Resistance force} \times v_d$$

$$T \times 2.14 = 500 \times 51.21 \times 10^{-3}; \quad T = 8.15 \text{ Nm}$$

Example 2.17

Determine all the instantaneous centres of the double slider-crank mechanism shown in Fig.2.35(a).



(a) Double slider–crank mechanism (b) Circle diagram

Fig.2.35 Locating instantaneous centres of the double slider-crank mechanism

■ **Solution**

Here $n = 6$ so that $N = \dfrac{6(6-1)}{2} = 15$

1	2	3	4	5	6
√	√	√	√	√	
12	23	34	45	56	
(13)	(24)	(35)	(46)		
√					
14	(25)	(36)			
(15)	(26)				
√					
16					

As shown in Fig.2.35(b), we have

(13) : 12, 23; 14, 34

(15) : 16, 56; 12, 25

(24) : 23, 34; 12, 14

(26) : 12, 16; 25, 56

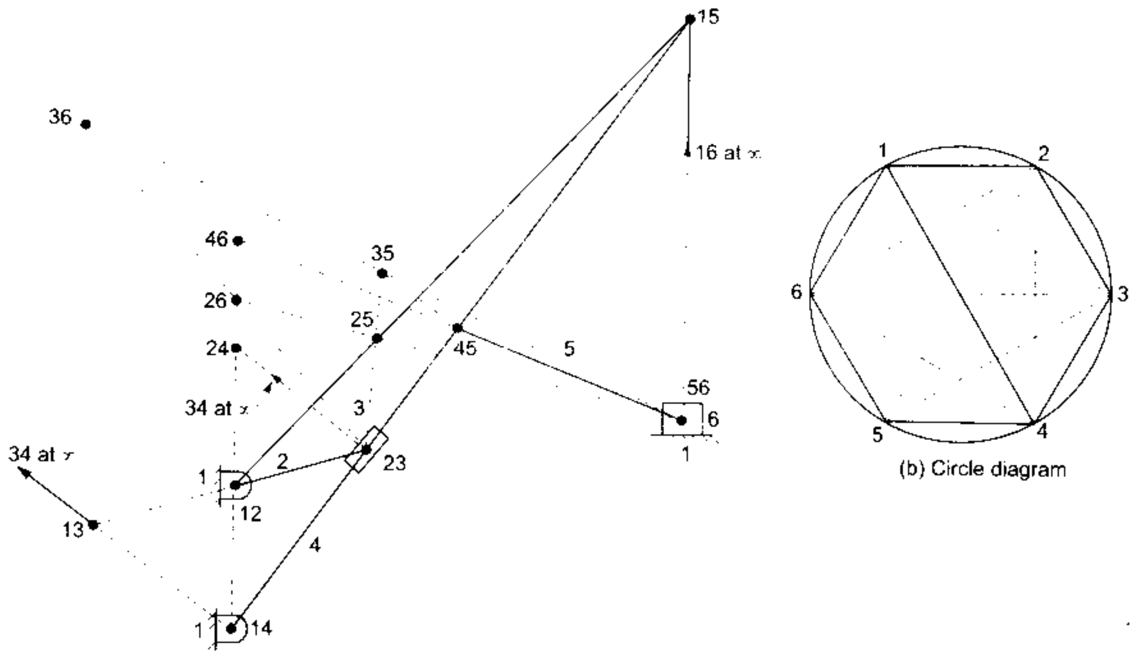
(36) : 13, 16; 35, 56

(45) : 14, 15; 34, 35

(46) : 45, 56; 34, 36

Example 2.18

Locate all the instantaneous centres of the Whitworth mechanism shown in Fig.2.36(a).



(a) Withworth mechanism

(b) Circle diagram

Fig.2.36 Locating instantaneous centres of the whitworth mechanism

■ **Solution** $N = \dfrac{6(6-1)}{2} = 15$

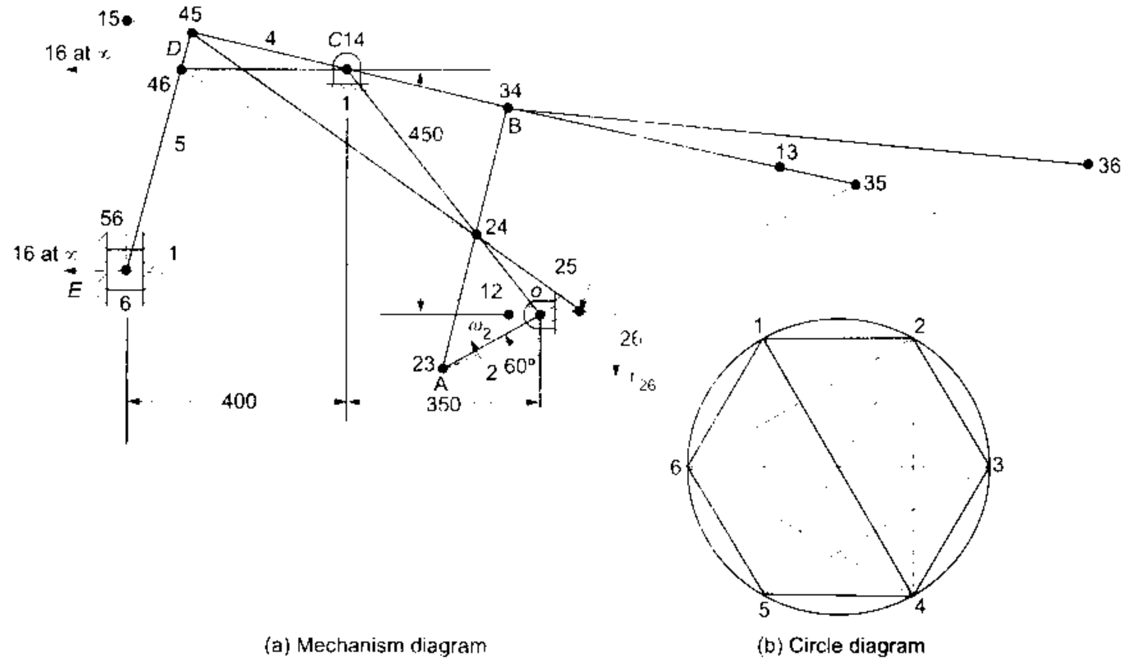
1	2	3	4	5	6
√	√	√	√	√	
12	23	34	45	56	
(13)	(24)	(35)	(46)		
√					
14	(25)	(36)			
(15)	(26)				
√					
16					

As shown in Fig.2.36(b), we have

13 : 12, 23; 14, 34
15 : 14, 45; 16, 56
24 : 23, 34; 12, 14
25 : 24, 45; 12, 15
26 : 25, 56; 12, 16
35 : 34, 45; 23, 25
36 : 35, 56; 23, 26
46 : 45, 46; 14, 16

Example 2.19

Determine all the instantaneous centres of the mechanism shown in Fig.2.37(a). Calculate the velocities of the slider *E* and the joints *B* and *D* when the crank *OA* is rotating at 120 rpm. Also find ω_{AB}, ω_{BD} and ω_{DE}. $OA = 200$ mm, $AB = 500$ mm, $BC = 300$ mm, $BD = 600$ mm and $DE = 450$ mm.



(a) Mechanism diagram (b) Circle diagram

Fig.2.37 Locating instantaneous centres of a mechanism

■ Solution

$$\omega_2 = 2\pi \times \frac{120}{60} = 12.57 \text{ rad/s}$$

$$v_a = \omega_2 \cdot OA = 12.57 \times 0.2 = 2.513 \text{ m/s}$$

$$N = \frac{6(6-1)}{2} = 15$$

1	2	3	4	5	6
✓12	✓23	✓34	✓45	✓56	
(13)	(24)	(35)	(46)		
✓14	(25)	(36)			
(15)	(26)				
✓16					

As shown in Fig.2.37(b), we have

13 : 12, 23; 14, 34

15 : 14, 45; 16, 56

24 : 23, 34; 12, 14

25 : 24, 45; 12, 15

26 : 25, 56; 12, 16

35 : 34, 45; 23, 25

36 : 35, 56; 23, 26

46 : 45, 46; 14, 16

$$v_{26} = \omega_2(12-26) = 12.57 \times 0.15 = 1.886 \text{ m/s}$$

$$v_e = v_{26} = 1.886 \text{ m/s}$$

$$13 - A = 710 \text{ mm},\ 13 - B = 510 \text{ mm},\ 14 - B = 300 \text{ mm},$$

$$14 - D = 300 \text{ mm},\ 15 - D = 110 \text{ mm},\ 15 - E = 470 \text{ mm}$$

$$v_b = \left[\frac{13 - B}{13 - A}\right] v_a = 510 \times \frac{2.513}{710} = 1.805 \text{ m/s}$$

$$v_d = \left[\frac{14 - C}{14 - B}\right] v_b = 300 \times \frac{1.805}{300} = 1.805 \text{ m/s}$$

$$v_e = \left[\frac{15 - E}{15 - D}\right] v_d = 470 \times \frac{1.805}{110} = 7.712 \text{ m/s}$$

$$\omega_{AB} = \frac{v_a}{(13 - A)} = \frac{2.513}{0.71} = 2.54 \text{ rad/s}$$

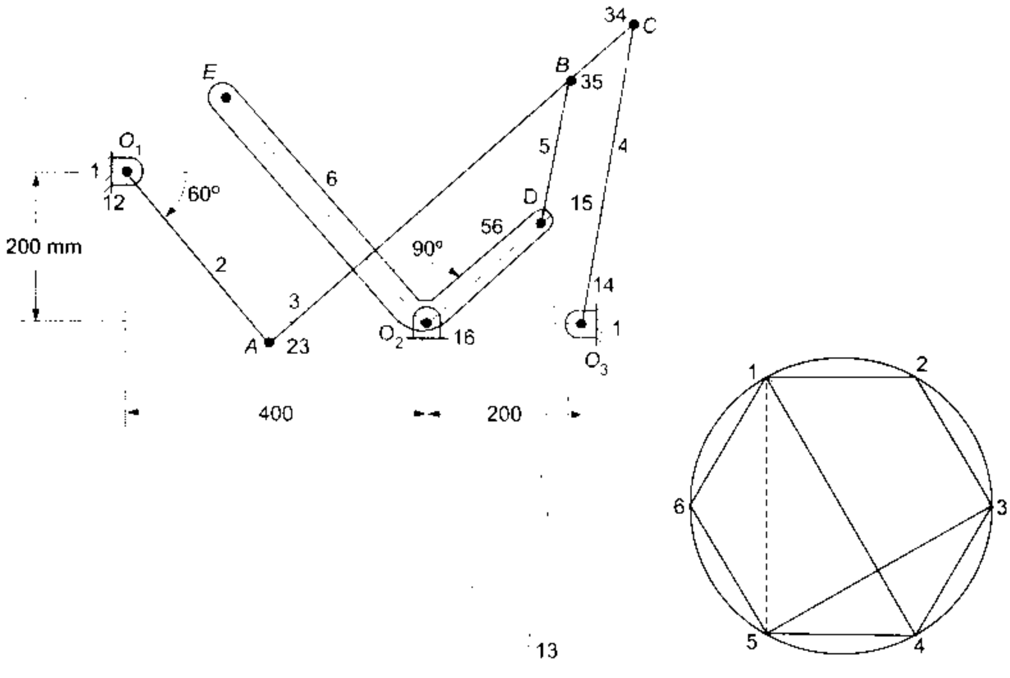
$$\omega_{BD} = \frac{v_b}{(14 - B)} = \frac{1.805}{0.3} = 6.02 \text{ rad/s}$$

$$\omega_{DE} = \frac{v_d}{(15 - D)} = \frac{1.805}{0.11} = 16.41 \text{ rad/s}$$

Example 2.20

A wrapping mechanism is shown in Fig.2.38(a). The crank O_1A rotates at a uniform speed of 1200 rpm. Determine the velocity of point E on the bell crank lever.

$$O_1A = 300 \text{ mm}, AC = 650 \text{ mm}, BC = 100 \text{ mm}, O_3C = 400 \text{ mm},$$
$$O_2E = 400 \text{ mm}, O_2D = 200 \text{ mm} \quad \text{and} \quad BD = 200 \text{ mm}.$$



(a) (b)

Fig.2.38 Wrapping mechanism

■ **Solution**

$$\omega = 2\pi \times \frac{1200}{60} = 125.7 \text{ rad/s}$$
$$v_a = 125.7 \times 0.3 = 37.71 \text{ m/s}$$
$$N = \frac{6(6-1)}{2} = 15$$

As shown in Fig.2.38(b), we have

1	2	3	4	5	6
√	√	√	√	√	
12	23	34	45	56	
		√			
(13)	24	35	46		
√					
14	25	36			
(15)	26				
√					
16					

13 : 12, 23; 14, 34

15 : 14, 45; 16, 56

$$13 - A = 530 \text{ mm}, 13 - B = 750 \text{ mm}, 15 - B = 170 \text{ mm},$$

$$15 - D = 30 \text{ mm}, 16 - D = 200 \text{ mm}, 16 - E = 400 \text{ mm}$$

$$v_b = \left[\frac{13 - B}{13 - A}\right] v_a = 750 \times \frac{37.71}{530} = 52.36 \text{ m/s}$$

$$v_d = \left[\frac{15 - D}{15 - B}\right] v_b = 30 \times \frac{52.36}{170} = 9.42 \text{ m/s}$$

$$v_e = \left[\frac{16 - E}{16 - D}\right] v_d = 400 \times \frac{9.42}{200} = 18.83 \text{ m/s}$$

Example 2.21

The sewing machine needle bar mechanism is shown in Fig.2.39(a). Crank 2 rotates at 450 rpm. Determine the velocity of the needle at D.

$O_1A = 15$ mm, $O_2B = 25$ mm, $AB = 65$ mm, $BC = 20$ mm, $CD = 60$ mm and $\angle O_2BC = 90°$.

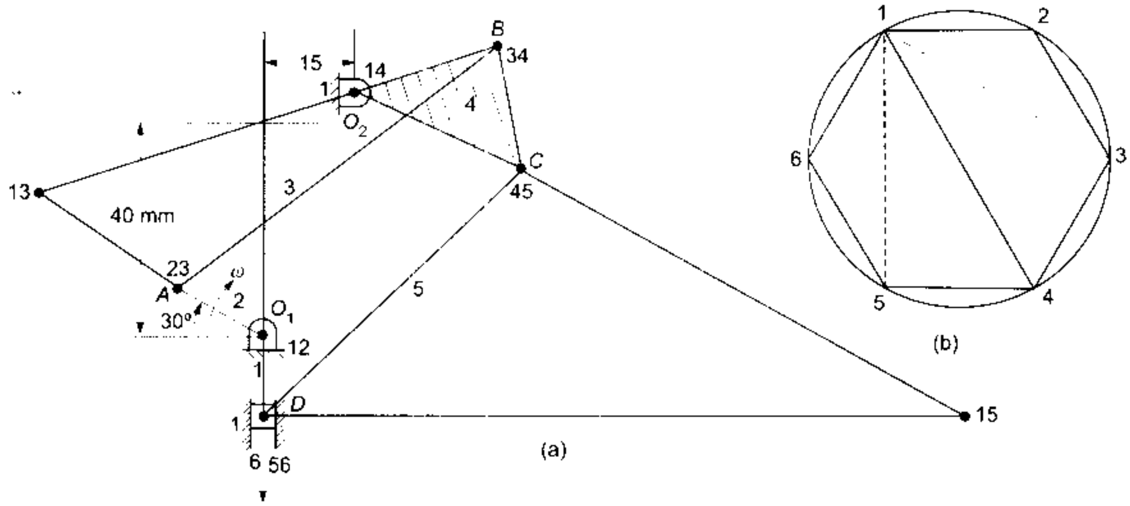


Fig.2.39 Sewing machine needle bar mechanism

■ Solution

Here $n = 6$ and $N = 15$

As shown in Fig.2.39(b), we have

1	2	3	4	5	6
√	√	√	√	√	
12	23	34	45	56	
(13)	24	35	46		
√					
14	25	36			
(15)	26				
√					
16					

$$13 : 12, 23; 14, 34$$
$$15 : 14, 45; 16, 56$$
$$13 - A = 32 \text{ mm}, 13 - B = 84 \text{ mm}, 14 - B = 25 \text{ mm},$$
$$14 - C = 30 \text{ mm}, 15 - C = 98 \text{ mm}, 15 - D = 132 \text{ mm}$$
$$\omega = 2\pi \times \frac{450}{60} = 47.124 \text{ rad/s}$$
$$v_a = 47.124 \times 0.015 = 0.71 \text{ m/s}$$
$$v_b = \left[\frac{13 - B}{13 - A}\right] v_a = 84 \times \frac{0.71}{32} = 1.864 \text{ m/s}$$
$$v_c = \left[\frac{14 - C}{14 - B}\right] v_b = 30 \times \frac{1.864}{25} = 2.236 \text{ m/s}$$
$$v_d = \left[\frac{15 - D}{15 - C}\right] v_c = 132 \times \frac{2.236}{98} = 3.01 \text{ m/s}$$

2.5 ACCELERATION DIAGRAMS

2.5.1 Total Acceleration of a Link

Consider two points A and B on a rigid link, as shown in Fig.2.40(a), such that point B moves relative to point A with an angular velocity ω and angular acceleration α.

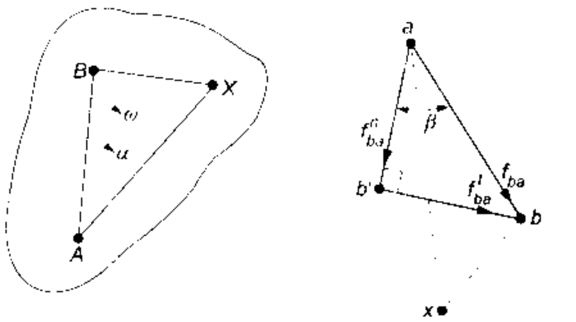


Fig.2.40 Acceleration for a link

Centripetal (or normal or radial) acceleration of point B with respect to point A is

$$f^n_{ba} = \omega^2 \cdot AB = \frac{v^2_{ba}}{AB} \tag{2.15}$$

Tangential acceleration of point B with respect to point A

$$f^t_{ba} = \alpha \cdot AB \tag{2.16}$$

Total acceleration of B with respect to A,

$$f_{ba} = f^n_{ba} + f^t_{ba} = \frac{v^2_{ba}}{AB} + \alpha \cdot AB$$
$$= (\omega^2 + \alpha) \cdot AB \tag{2.17}$$
$$\tan\beta = \frac{f^t_{ba}}{f^n_{ba}} = \frac{\alpha}{\omega^2} \tag{2.18}$$

The acceleration diagram has been represented in Fig.2.40(b). The + sign represents vectorial addition.

2.5.2 Acceleration of a Point on a Link

The accelerations of any point *X* on the rigid link with respect to *A* [Fig.2.40(a)] are

$$f^n_{xa} = \omega^2 \cdot AX \tag{2.19}$$

$$f^t_{xa} = \alpha \cdot AX \tag{2.20}$$

Total acceleration,

$$f_{xa} = f^n_{xa} + f^t_{xa} \tag{2.21}$$

Therefore, f_{xa} denoted by *ax* in the acceleration diagram [Fig.2.40(b)] is inclined to *XA* at the same angle β. Triangles *abx* and *ABX* are similar. Thus, point *x* can be fixed on the acceleration image, corresponding to point *X* on the link.

Total acceleration of *X* relative to *A*,

$$f_{xa} = ax$$

Total acceleration of *X* relative to *B*,

$$f_{xb} = bx$$

2.5.3 Absolute Acceleration for a Link

Consider the rigid link *AB* such that point *B* is rotating about *A* with angular velocity ω and angular acceleration α, as shown in Fig.2.41(a). The point *a* itself has acceleration f_a. The acceleration diagram is shown in Fig.2.41(b). The absolute acceleration of *B* is given by

$$\begin{aligned} f_b &= f_a + f^n_{ba} + f^t_{ba} \\ &= f_a + \omega^2 \cdot AB + \alpha \cdot AB \end{aligned} \tag{2.22}$$

Similarly, for any other point *X*,

$$\begin{aligned} f_x &= f_a + f^n_{xa} + f^t_{xa} \\ &= f_a + \omega^2 \cdot AX + \alpha \cdot AX \end{aligned} \tag{2.23}$$

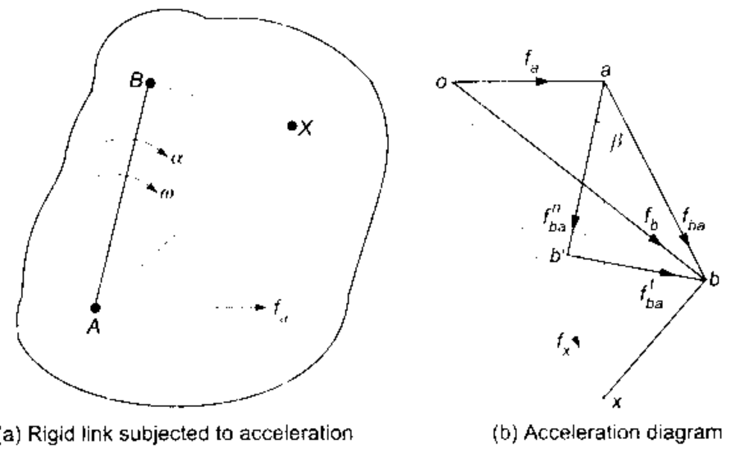
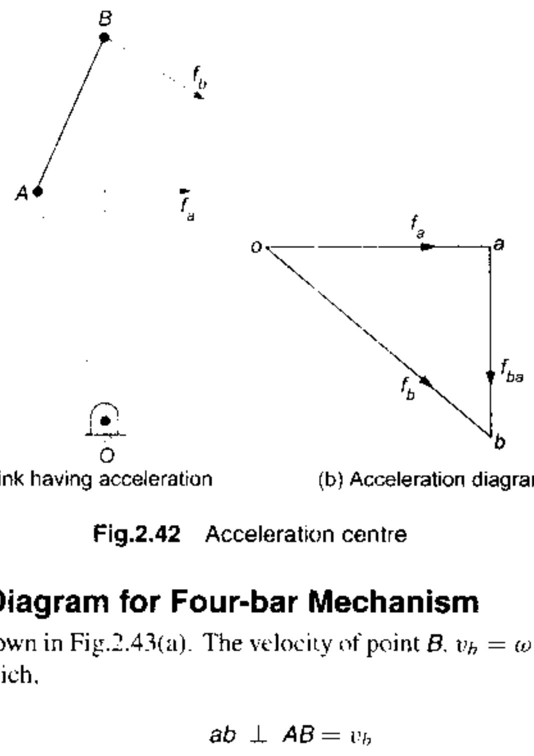


Fig.2.41 Absolute acceleration of a rigid link

2.5.4 Acceleration Centre

Consider a rigid link *AB* whose ends *A* and *B* have accelerations f_a and f_b, respectively, as shown in Fig.2.42(a). The acceleration diagram is shown in Fig.2.42(b). If we select a point *O* on the link such that triangles *aob* and *AOB* are similar, then the acceleration of point *O* relative to a fixed link or fixed point *O* is zero. The point *O* is called the instantaneous centre of acceleration of link *AB* or *acceleration centre*.



(a) Link having acceleration (b) Acceleration diagram

Fig.2.42 Acceleration centre

2.5.5 Acceleration Diagram for Four-bar Mechanism

The four-bar mechanism is shown in Fig.2.43(a). The velocity of point *B*, $v_b = \omega \cdot AB$. The velocity diagram is shown in Fig.2.43(b), in which,

$$ab \perp AB = v_b$$
$$dc \perp DC$$
and
$$bc \perp BC$$
$$ac = v_c$$
$$bc = v_{cb}$$
Now
$$f_b = \frac{v_b^2}{AB} = ab$$
$$f_{cb}^n = \frac{v_{cb}^2}{BC} = bc'$$
$$f_{cd}^n = \frac{v_c^2}{CD} = dc''$$
$$f_{cb} = f_{cb}^n + f_{cb}^t$$
$$bc = bc + c'c$$
$$f_{cd} = f_{cd}^n + f_{cd}^t$$
$$dc = dc' + c''c$$

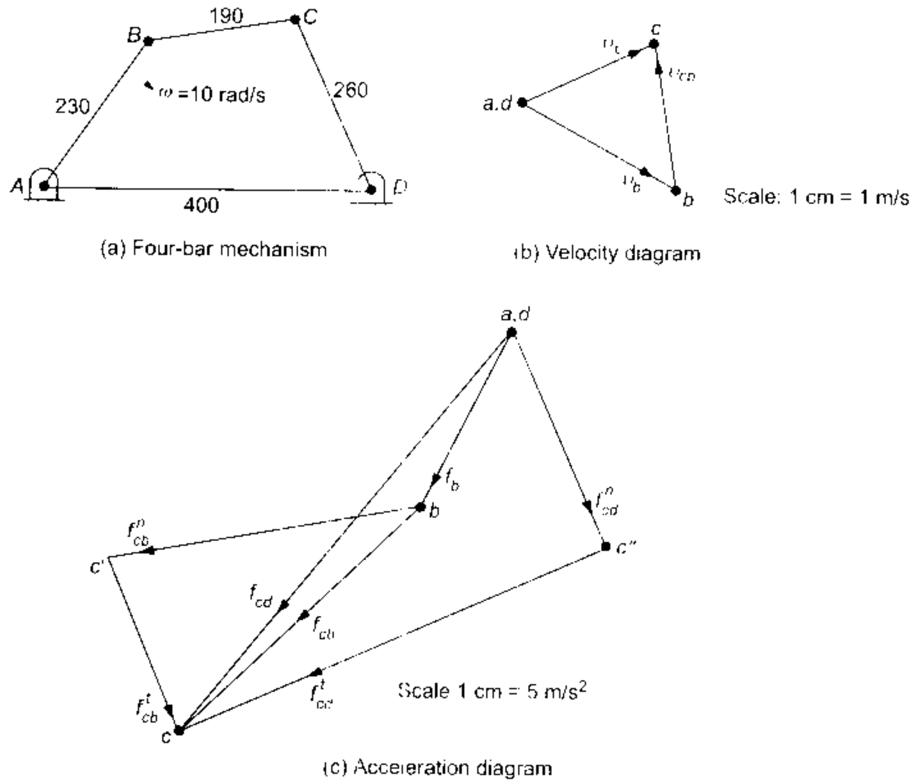


Fig.2.43 Acceleration diagram acceleration diagram for a four-bar mechanism

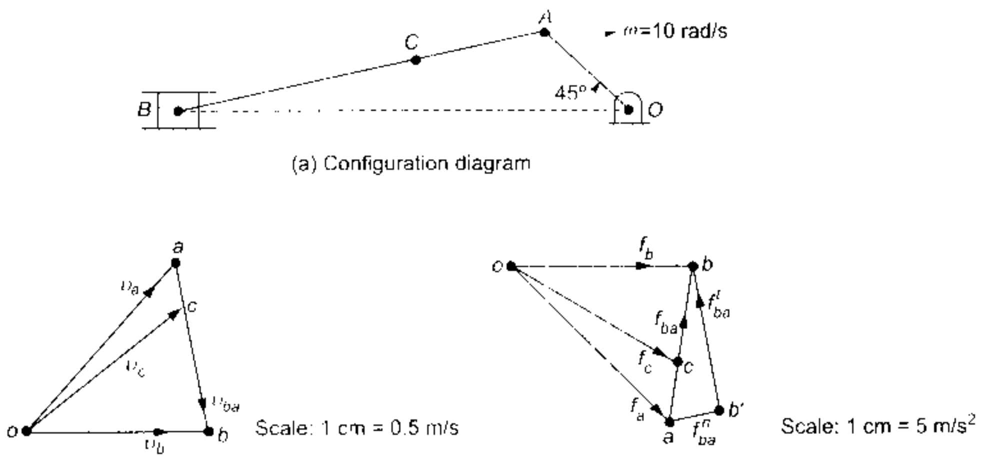
The acceleration diagram is shown in Fig.2.43(c). To construct the acceleration diagram, proceed as follows:

1. Draw $ab = f_b$ parallel to AB. Draw $bc' = f^n_{cb}$ parallel to BC. f^n_{cb} is known in magnitude and direction.
2. Draw cc', representing the f^t_{cb}, perpendicular to bc'. However, f^t_{cb} is known in direction only.
3. Now draw $dc'' = f^n_{cd}$ parallel to CD, which is known in magnitude and direction.
4. Draw $c''c$, representing f^t_{cd}, perpendicular to dc'', to intersect $c'c$ at c. Join dc and bc. Then $bc = f_{cb}$ and $dc = f_{cd}$.

2.5.6 Acceleration Diagram for the Slider–crank Mechanism

The slider–crank mechanism is shown in Fig.2.44(a). Here, $v_a = \omega \cdot OA$. The velocity diagram is shown in Fig.2.44(b), in which

$$oa \perp OA = v_a$$
$$ab \perp AB$$
$$ob \parallel OB$$
$$ob = v_b$$
$$ab = v_{ba}$$



(a) Configuration diagram

(b) Velocity diagram

(c) Acceleration diagram

Fig.2.44 Acceleration diagram for slider-crank mechanism

The acceleration diagram is shown in Fig.2.44(c), in which

$$f_a = \omega^2 \cdot OA = \frac{v_a^2}{OA} = oa$$

$$f^n_{ba} = \frac{v_{ba}^2}{AB} = ab'$$

Draw $oa = f_a$ parallel to OA. Draw $ab' = f^n_{ba}$ parallel to AB and $bb' \perp ab'$ to represent f^t_{ba}. f^t_{ba} is known in direction only. Draw ob parallel to OB to intersect $b'b$ at b. Join ab. Then,

$$ab = f_{ba}$$

$$ob = f_b = \text{linear acceleration of slider } B$$

To find the acceleration of any point C in AB, we have

$$\frac{ac}{ab} = \frac{AC}{AB}$$

Join oc. Then $f_c = oc$.

2.5.7 Four-bar Mechanism with Ternary Link

The four-bar mechanism with a ternary link is shown in Fig.2.45(a), in which the driving crank has angular velocity ω and angular acceleration $\alpha \cdot v_a = \omega \cdot O_1A$. The velocity diagram is shown in Fig.2.45(b).

$$o_1a \perp O_1A = v_a$$

$$ab \perp AB$$

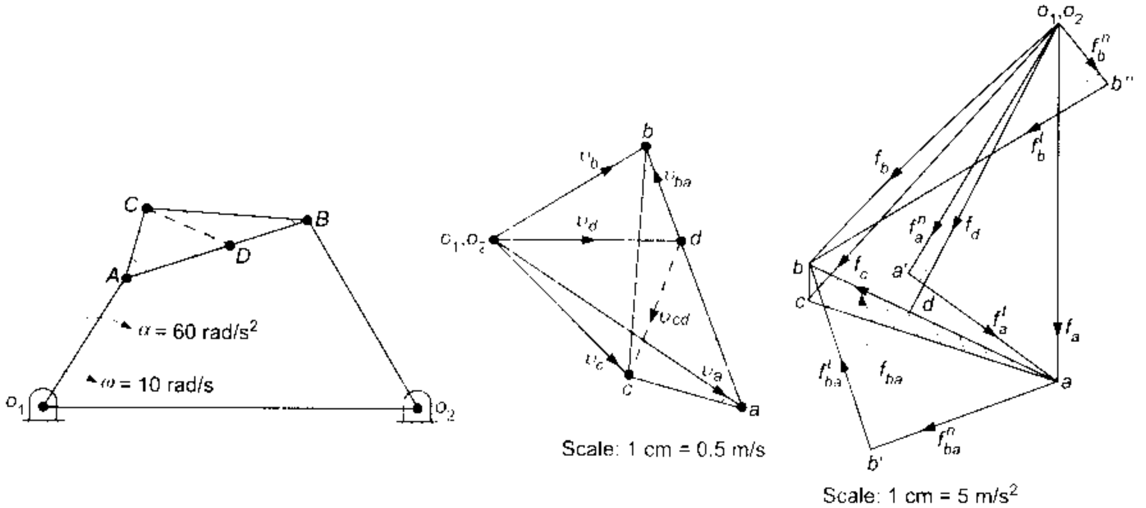
$$o_2b \perp O_2B = v_b$$

$$ab = v_{ba}$$

$$bc \perp BC$$

$$ac \perp AC$$

$$\frac{ad}{ab} = \frac{AD}{AB}$$



(a) Configuration diagram (b) Velocity diagram (c) Acceleration diagram

Fig.2.45

Join *cd*, then *abc* is the velocity image of ternary link *ABC*.

$$f^n_{ao1} = \frac{v_a^2}{O_1A} = o_1a'$$

$$f^t_{ao1} = \alpha \cdot O_1A = a'a$$

$$f_{ao1} = f^n_{ao1} + f^t_{ao1} = o_1a$$

$$f^n_{bo2} = \frac{v_b^2}{O_2B} = o_2b''$$

$$f^n_{ba} = \frac{v_{ba}^2}{AB} = ab'$$

$$bb' = f^t_{ba} \perp ab'$$

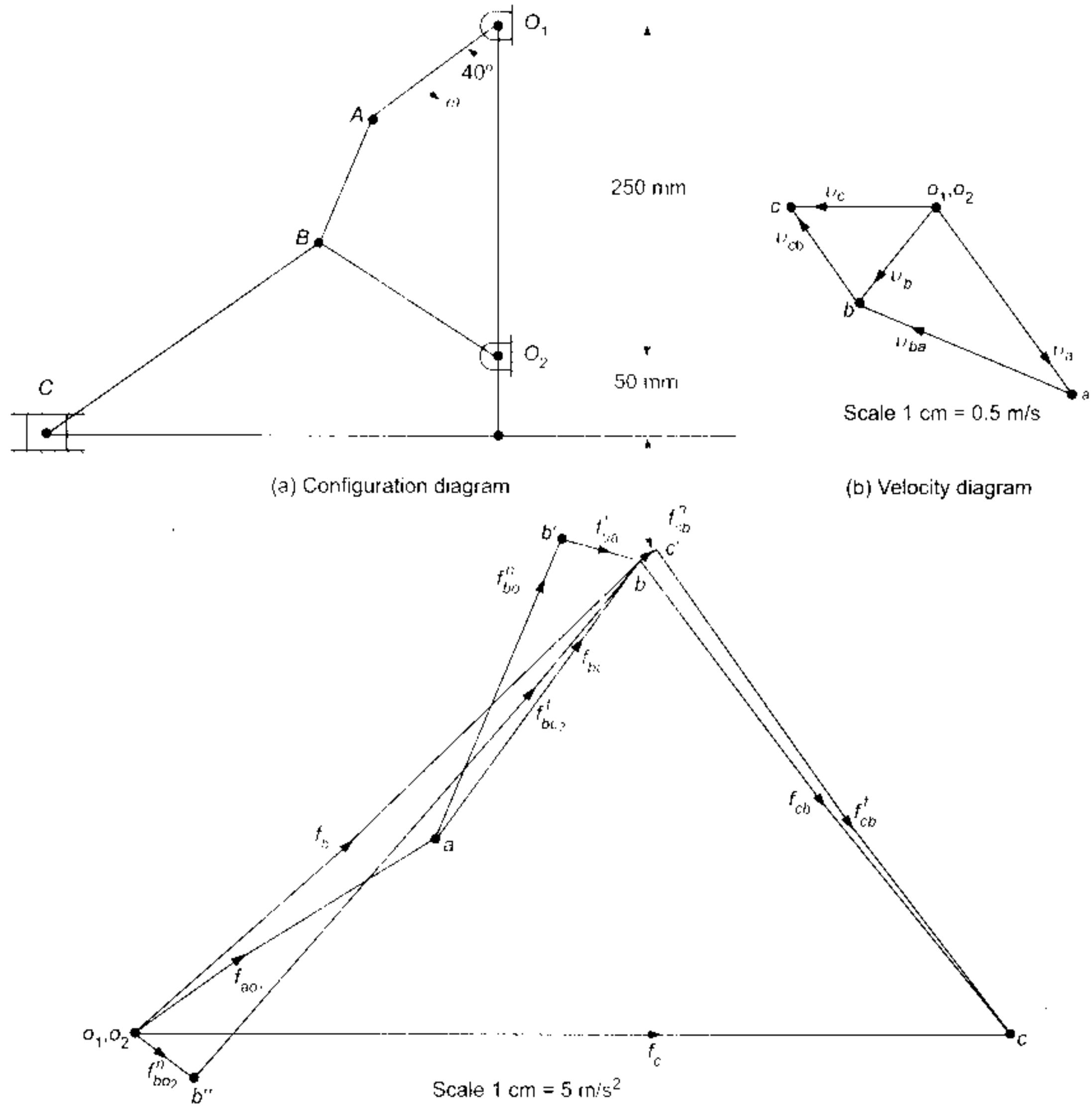
$$bb'' = f^t_{bo2} \perp o_2b''$$

$$f_{bo2} = f_{ao1} + f_{ba} = f^n_{ao1} + f^t_{ao1} + f^n_{ba} + f^t_{ba}$$

Join o_1d, then $o_1d = f_d$. Draw $ac \perp AC$ and $bc \perp BC$. Join o_1c. Then $o_1c = f_c$. Triangles *abc* and *ABC* are similar. Thus, *abc* is the acceleration image of ternary link *ABC*. The acceleration diagram is shown in Fig.2.45(c).

Example 2.22

In the mechanism shown in Fig.2.46(a), determine the acceleration of the slider C. O_1A = 100 mm. AB = 105 mm. O_2B = 150 mm and BC = 300 mm. Crank O_1A rotates at 180 rpm.



(a) Configuration diagram

(b) Velocity diagram

(c) Acceleration diagram

Fig.2.46

■ Solution

$$\omega = 2\pi \times 150/60 = 15.7 \text{ rad/s}$$

$$v_a = \omega \cdot O_1A = 15.7 \times 0.1 = 1.57 \text{ m/s}$$

Draw the velocity diagram [Fig.2.46(b)] to a scale of 1 cm = 0.5 m/s, in which

$$o_1a \perp O_1A = v_a$$

$$ab \perp AB$$
$$o_2b \perp O_2B$$
$$o_2b = v_b = 1.7 \text{ cm} = 0.85 \text{ m/s}$$
$$ab = v_{ba} = 3 \text{ cm} = 1.5 \text{ m/s}$$
$$bc \perp BC$$
$$o_1c \parallel CD$$
$$o_1c = v_c = 1.9 \text{ cm} = 0.95 \text{ m/s}$$
$$bc = v_{cb} = 1.5 \text{ cm} = 0.75 \text{ m/s}$$

Draw the acceleration diagram [Fig.2.46(e)] to a scale of 1 cm = 5 m/s^2.

$$f_{ao1} = \frac{v_a^2}{O_1A} = oa = \frac{(1.57)^2}{0.1} = 24.65 \text{ m/s}^2$$
$$f_{ba}^n = \frac{v_{ba}^2}{AB} = ab' = \frac{1.5^2}{0.105} = 21.43 \text{ m/s}^2$$
$$f_{ba}^t = b'b \perp ab'$$
$$f_{bo2}^n = \frac{v_b^2}{O_2B} = o_2b'' = \frac{0.85^2}{0.15} = 4.82 \text{ m/s}^2$$
$$f_{bo2}^t = b''b \perp o_2b''$$
$$o_2b = f_b \quad \text{and} \quad ab = f_{ba}$$
$$f_{cb}^n = \frac{v_{cb}^2}{BC} = bc' = \frac{0.75^2}{0.3} = 1.875 \text{ m/s}^2$$
$$f_{cb}^t = c'c \perp bc'$$

$o_1c \parallel CD$. Then $o_1c = f_c$, $bc = f_{cb}$.
Acceleration of slider C, f_c = 11.5 cm = 57.5 m/s^2.

2.6 CORIOLIS ACCELERATION

It has been observed in Section 2.5 that the total acceleration of a point with respect to another point in a rigid link is the vector sum of its normal and tangential components. This holds true when the distance between the two points is fixed and the relative acceleration of the two points on a moving rigid link has been considered. If the distance between the two points varies, that is, if the second point which was considered stationary, now slides, the total acceleration will contain one additional component called the Coriolis acceleration component.

Consider slider B on link OA such that when link OA is rotating clockwise with angular velocity ω the slider B moves outward with linear velocity v, as shown in Fig.2.47. Let the angle turned through by link OA in time dt in order to occupy the new position OA' be $d\theta$ and let the slider move to position E. The slider can be considered to move from B to E as follows:

1. From B to C due to the angular velocity ω of link OA.
2. C to D due to the outward velocity v of the slider.
3. D to E due to the acceleration perpendicular to the rod, that is, due to Coriolis acceleration.

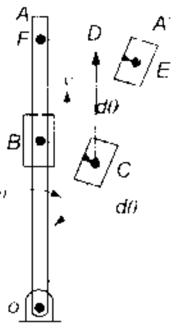


Fig.2.47 Slider B on a rotating link OA

Now,

$$\begin{aligned}
\text{arc } DE &= \text{arc } EF - \text{arc } FD \\
&= \text{arc } EF - \text{arc } BC \\
&= OF \cdot d\theta - OB \cdot d\theta \\
&= (OF - OB)d\theta \\
&= BF \cdot d\theta = CD \cdot d\theta
\end{aligned}$$

Now, $$CD = v \cdot dt$$

and $$d\theta = \omega \cdot dt$$

Hence, $$\text{arc } DE = (v \cdot dt) \cdot (\omega \cdot dt)$$

Now, $$\text{arc } DE = \text{chord } DE, \text{ as } d\theta \text{ is very small.}$$

Therefore, $$DE = v \cdot \omega (dt)^2 \tag{2.24}$$

But, $$DE = \frac{1}{2} \cdot f^{cr} (dt)^2 \tag{2.25}$$

where f^{cr} is the constant Coriolis acceleration of the particle.

Hence, from (2.24) and (2.25) we get

$$f^{cr} = 2v\omega \tag{2.26}$$

The direction of the Coriolis acceleration component is such as to rotate the sliding velocity vector in the same sense as the angular velocity of OB. This is achieved by turning the sliding velocity vector through 90° in a manner that the velocity of this vector is the same as that of angular velocity of OA. The method of finding the direction of Coriolis acceleration is illustrated in Fig.2.48.

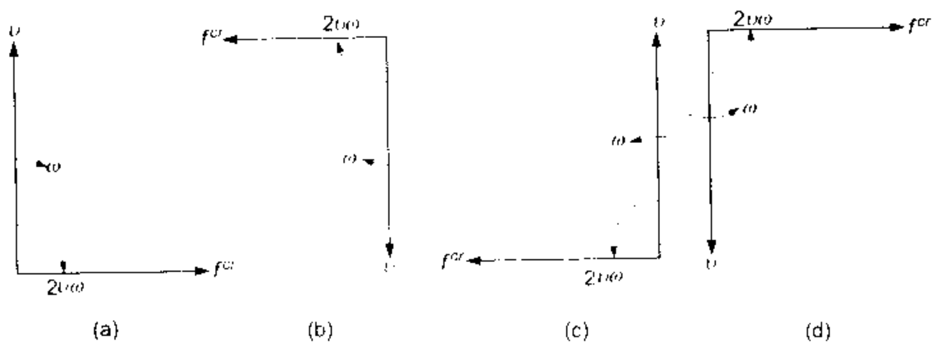


Fig.2.48 Finding direction of coriolis acceleration

Example 2.23

In the crank and slotted lever type quick-return motion mechanism, shown in Fig.2.49(a), crank AB rotates at one rev/s. Determine (a) the velocity of the ram at D, (b) the magnitude of the Coriolis acceleration component and (c) the acceleration of the ram at D. $AB = 200$ mm, $OC = 600$ mm and $CD = 500$ mm and $OA = 250$ mm.

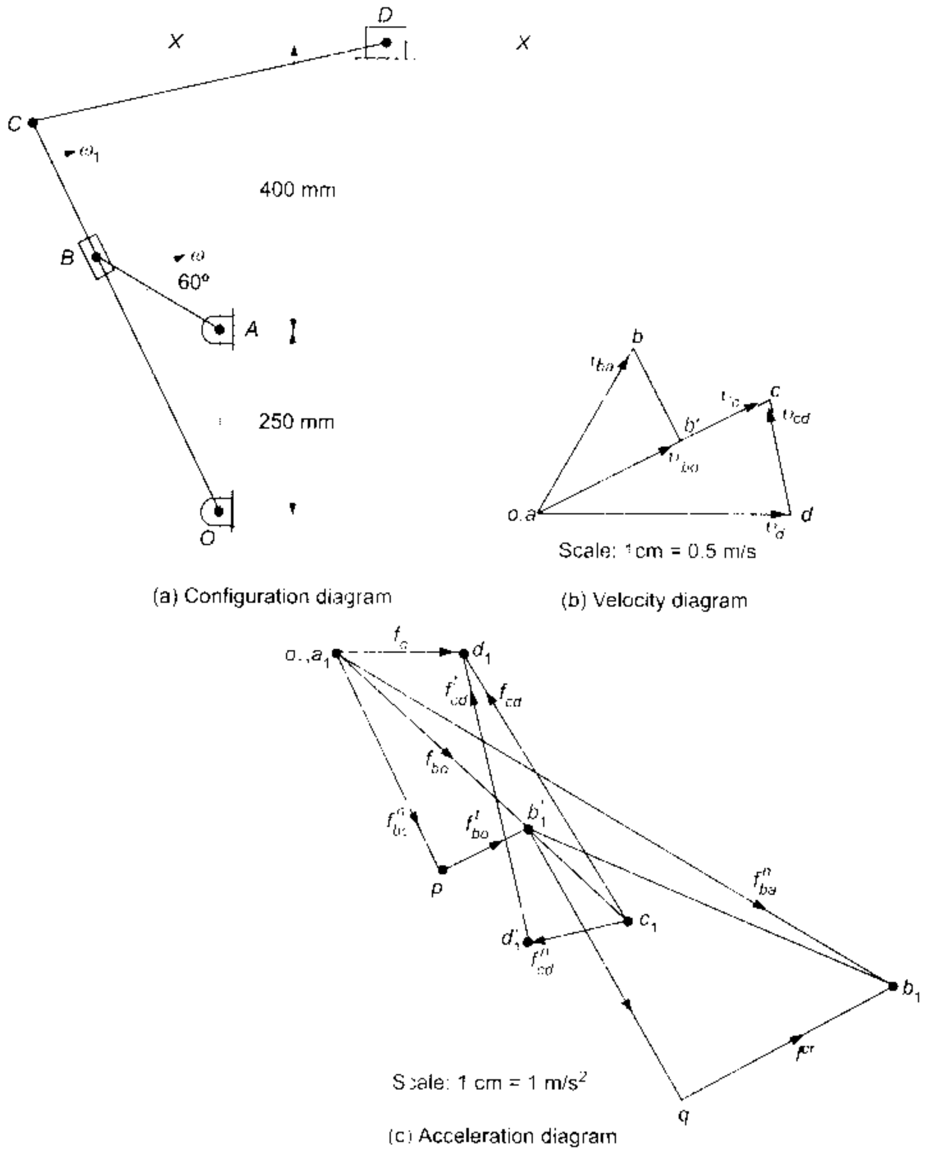


Fig.2.49

■ Solution

$$\omega = 2\pi \times 1 = 6.28 \text{ rad/s}$$

$$v_b = v_{ba} = \omega \cdot AB = 6.28 \times 0.2 = 1.256 \text{ m/s}$$

The velocity diaram is shown in Fig.2.49(b) to a scale of 1 cm = 0.5 m/s, in which, $ba \perp AB = 0.628$ m/s, $oc \perp OB$, $bb' \parallel OB$, $oc = ob'$ (OC/OB),

$od \parallel XX$ and $cd \perp CD$.

$$ob' = v_{bo} = 2.3 \text{ cm} = 1.15 \text{ m/s}$$

$$oc = ob'(OC/OB)$$

$$= \frac{2.3 \times 600}{390} = 3.54 \text{ mm}$$

$$v_c = v_{co} = oc = 1.77 \text{ m/s}$$

Velocity of slider $B = bb' = 1.5 \text{ cm} = 0.75$ m/s

Velocity of ram D. $v_d = ad = 3.6 \text{ cm} = 1.8$ m/s

Angular velocity of lever OBC. $\omega_1 = \dfrac{ob'}{OB}$

$$= \frac{1.15}{0.39} = 2.95 \text{ rad/s (cw)}$$

$$v_{cd} = dc = 1.7 \text{ cm} = 0.85 \text{ m/s}$$

Coriolis acceleration of $B = 2bb'$. $\omega_1 = 2 \times 1.5 \times 0.5 \times 2.95 = 4.425$ m/s²

Table 2.1 shows the calculation for Coriolis acceleration.

Table 2.1

Link	Length r	Velocity v, m/s	Normal acceleration v^2/r, m/s²	Tangential acceleration m/s²	Coriolis acceleration m/s²
AB	0.2	1.256	7.89	–	–
OB	0.39	1.15	2.39	–	–
Slider B	–	–	–	–	4.425 ⊥ OB
CD	0.5	0.85	1.445	–	–

The acceleration diagram is shown in Fig.2.49(c) to a scale of 1 cm = 1 m/s², in which

$$a_1b_1 \parallel AB = f^n_{ba} = 7.89 \text{ m/s}^2$$

$$b_1q = f^{cr} = 4.425 \text{ m/s}^2 \perp OB$$

$$qb'_1 \perp qb_1$$

$$o_1p = f^n_{bo} = 2.39 \text{ m/s}^2 \parallel OB$$

$$pb'_1 \perp o_1p$$

Join $b_1b'_1$ and $o_1b'_1 \cdot o_1b'_1 = 1.7$ cm. Produce O_1b_1 to c_1, such that

$$oc_1 = ob'_1\left(\frac{OC}{OB}\right)$$

$$= \frac{3.6 \times 600}{390} = 5.538 \text{ cm}$$

$$c_1d'_1 \parallel CD = f^n_{cd} = 1.445 \text{ m/s}^2$$

$$d'_1 d_1 \perp CD = f^t_{cd}$$
$$a_1 d_1 \parallel XX = f_d$$
$$c_1 d_1 = f_{cd} = 4.3 \text{ cm} = 4.3 \text{ m/s}^2$$
$$o_1 d_1 = f_d = 1.8 \text{ cm} = 1.8 \text{ m/s}^2$$

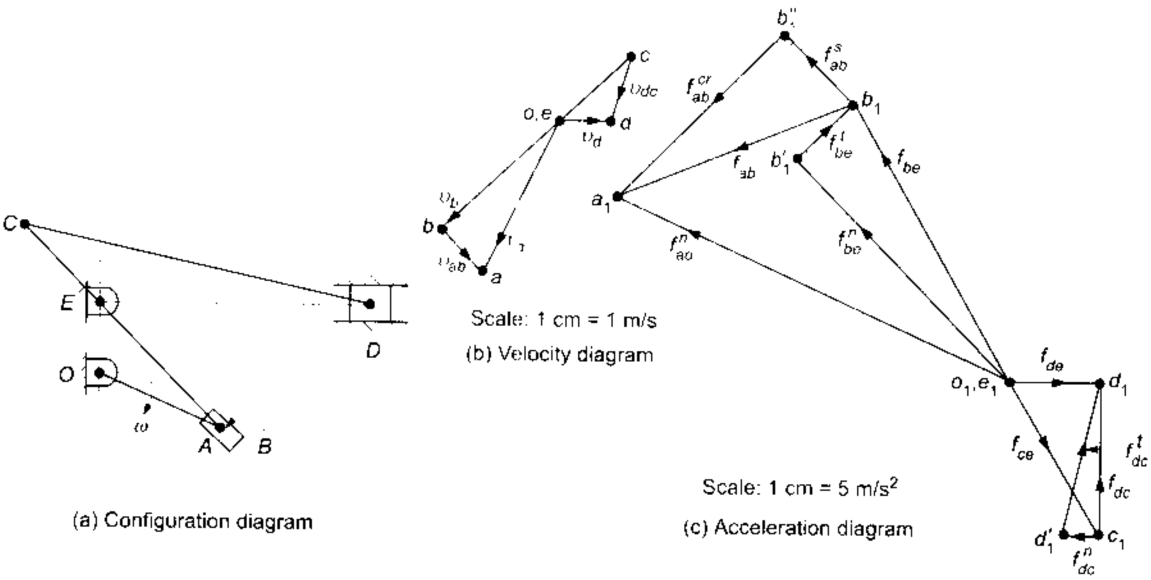
(a) $v_d = 1.8$ m/s

(b) $f^{cr} = 4.425$ m/s^2

(c) $f_d = 1.8$ m/s^2.

Example 2.24

Draw the acceleration diagram for the Whitworth mechanism shown in Fig.2.50(a). $OB = 200$ mm, $BE = 260$ mm, $BC = 410$ mm, $CD = 500$ mm and $OE = 100$ mm.



(a) Configuration diagram

(b) Velocity diagram

(c) Acceleration diagram

Fig.2.50

■ **Solution**

$$\omega = 2\pi \times \frac{120}{60} = 12.57 \text{ rad/s}$$
$$v_a = 12.57 \times 0.2 = 2.51 \text{ m/s}$$

The velocity diagram is shown in Fig.2.50(b), in which

$$oa = v_a \perp OA = 2.51 \text{ m/s}$$
$$ab \parallel BE$$
$$ob \perp BE$$
$$v_b = eb = 2.4 \text{ m/s}$$

$$\omega_{EB} = \frac{eb}{EB} = 2.4/0.26 = 9.23 \text{ rad/s (cw)}$$

$$bc = be\left(\frac{BC}{BE}\right) = 2.4\left(\frac{410}{260}\right) = 3.78 \text{ m/s}$$

$$v_d = ed = 0.95 \text{ m/s}$$

$$\omega_{CD} = \frac{cd}{CD} = \frac{0.95}{0.5} = 1.9 \text{ rad/s (cw)}$$

$$v_{ab} = ba = 0.9 \text{ m/s}$$

Table 2.2 shows the calculation of Coriolis acceleration.

Table 2.2

Link	Length r m	Velocity v m/s	Normal acceleration v^2/r m/s^2	Tangential acceleration $\alpha \cdot r$ m/s^2	Coriolis acceleration $2v\omega$ m/s^2
OA	0.2	2.51	31.5	–	–
BE	0.26	2.4	22.15	–	–
CD	0.50	0.95	1.80	–	–
Point *A* or *B*	–	0.90	–	–	$2 \times 0.9 \times 9.23$ $= 16.61$

The Coriolis acceleration of $AB = f_{ab}^{cr} = 2v_{ab}\omega_{EB}$

$$= 2 \times 0.9 \times 9.23 = 16.61 \text{ m/s}^2$$

The acceleration diagram is shown in Fig.2.50(c), in which

$$o_1a_1 = f_{ao}^n \parallel AO = 31.5 \text{ m/s}^2$$

$$o_1b_1' = f_{be}^n \parallel BE = 22.15 \text{ m/s}^2$$

$$b_1'b_1 = f_{be}^t \perp BE$$

$$a_1b_1'' \perp EB = f_{ab}^{cr} = 16.61 \text{ m/s}^2$$

$$b_1''b_1 \parallel EB = f_{ab}^s = \text{sliding acceleration of } A \text{ relative to } B$$

Locate b_1. Join a_1b_1 and o_1b_1.

$$\frac{c_1e_1}{c_1b_1} = \frac{CE}{CB}$$

$$c_1e_1 = \frac{150 \times 4.6}{260} = 2.65 \text{ cm}$$

Locate c_1.

$$c_1d_1' = f_{dc}^n = 1.80 \text{ m/s}^2$$

$$d_1'd_1 \perp c_1d_1'$$

$$e_1d_1 \parallel ED$$

Locate d_1. Join c_1d_1, then $c_1d_1 = f_{dc}$.

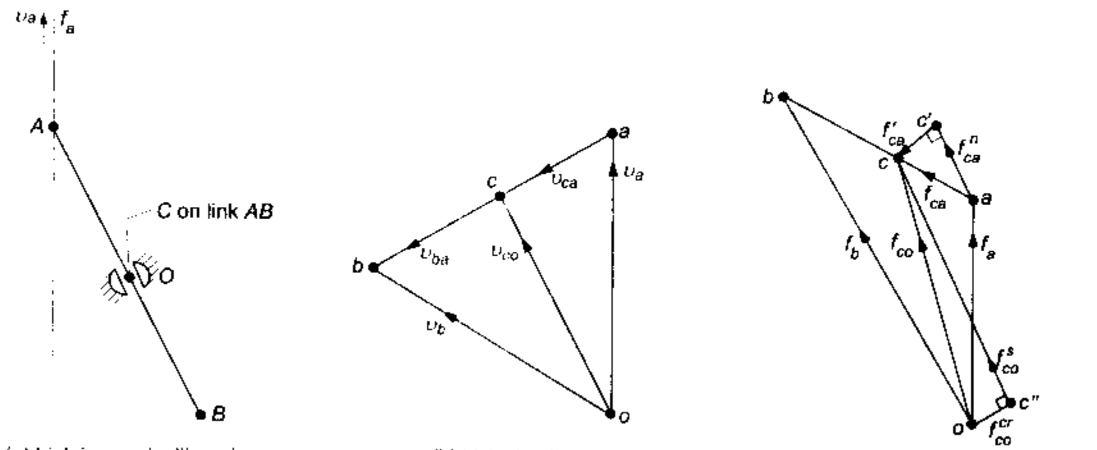
$$f_d = o_1d_1 = 1.3 \text{ cm} = 6.5 \text{ m/s}^2$$

Angular acceleration of EB,
$$\alpha_{EB} = \frac{f'_{eb}}{BE} = \frac{b'_1b_1}{BE} = \frac{1.2 \times 5}{0.26} = 23 \text{ rad/s}^2 \text{ (ccw)}$$

Angular acceleration of CD,
$$\alpha_{CD} = \frac{f'_{dc}}{CD} = \frac{d'_1d_1}{CD} = \frac{2.2 \times 5}{0.5} = 22 \text{ rad/s}^2 \text{ (ccw)}$$

2.7 LINK SLIDING IN A SWIVELLING PIN

Consider a link AB sliding through a swivelling pin O, as shown in Fig.2.51(a). Point C lies on link AB. The point A moves up with velocity v_a and acceleration f_a.



(a) Link in a awivelling pin (b) Velocity diagram (c) Acceleration diagram

Fig.2.51

Velocity diagram Draw $oa = v_a$ parallel to the path of motion of point A, as shown in Fig.2.51(b). From a draw ab perpendicular to AB to represent v_{ba}, and from o draw a line parallel to AB to represent v_{co}, the sliding velocity, to meet ab at c. Produce ac to b such that $\frac{ac}{ab} = AC/AB$. Join o to b. Then $ob = v_b$.

Acceleration diagram Draw f_a parallel to the path of A, as shown in Fig.2.51(c).

$$ac' = f^n_{ca} = \frac{v^2_{ca}}{CA} = \frac{ac^2}{CA \parallel AB}$$

$$f'_{ca} \perp ac'$$

$$oc'' = f^{cr}_{ca} = 2 \cdot \omega \cdot AB \cdot v_{co} \perp AB$$

$$co'' \parallel AB$$

Locate point c and join ac. Produce ac to b such that $ac/ab = AC/AB$. Join ob. Then $ob = f_b$.

Example 2.25

In the swivelling point mechanism shown in Fig.2.52(a). OA = 25 mm, AB = 150 mm, AD = DB, DE = 150 mm, EF = 100 mm, BC = 60 mm, DS = 40 mm and OC = 150 mm. Crank OA rotates at 200 rpm. Determine the acceleration of sliding link DE in the trunnion.

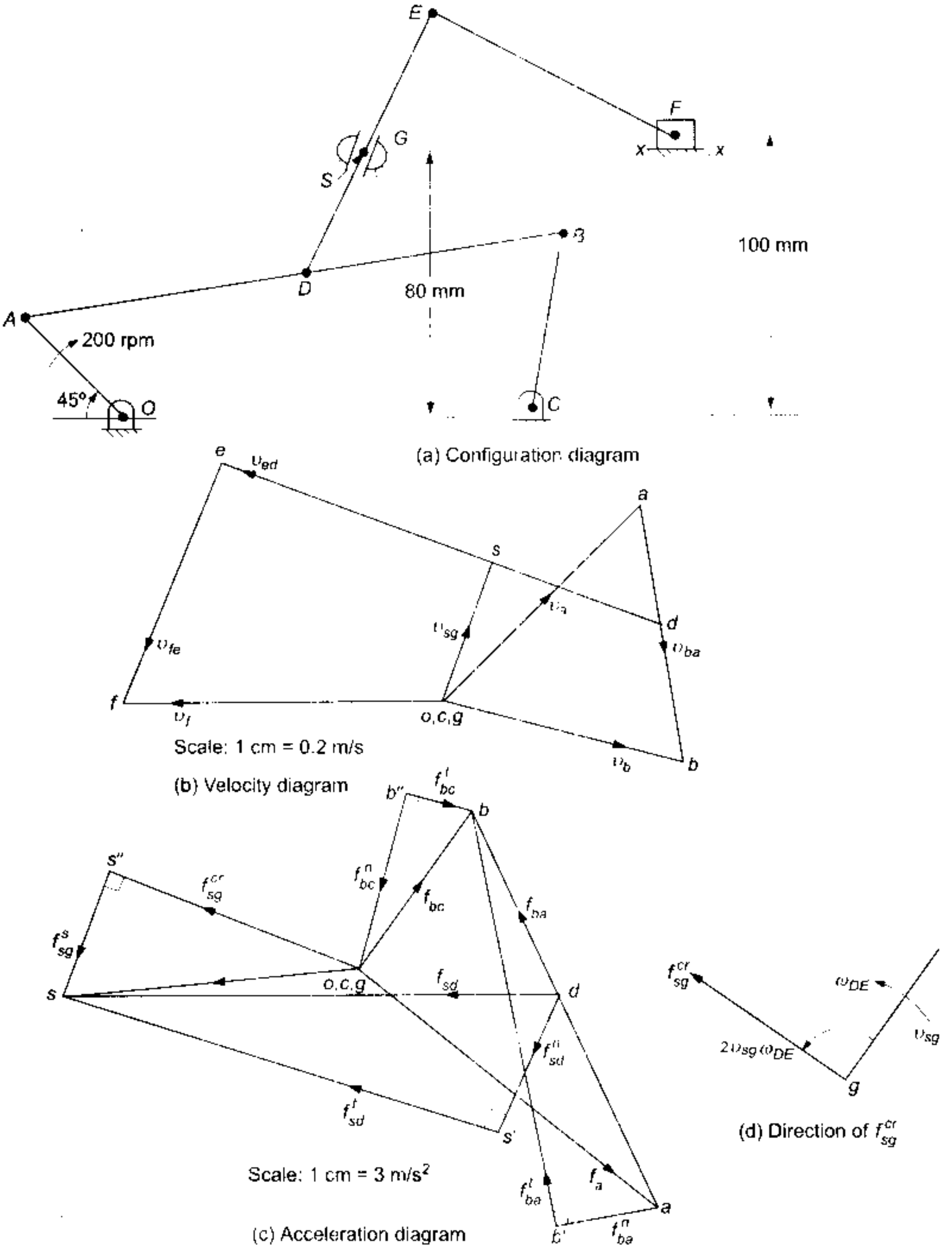


Fig.2.52

Solution

$$\omega = 2\pi \times 200/60 = 20.944 \text{ rad/s}$$
$$v_a = 20.944 \times 0.025 = 0.523 \text{ m/s}$$

The velocity diagram is shown in Fig.2.52(b) to a scale of 1 cm = 0.1 m/s, in which

$$oa = v_a \perp OA$$
$$ab \perp AB$$
$$cb \perp BC$$

Locate *b*.

$$cb = v_b = 4.6 \text{ cm} = 0.46 \text{ m/s}$$
$$ab = v_{ba} = 2.3 \text{ cm} = 0.23 \text{ m/s}$$
$$\frac{ad}{ab} = \frac{AD}{AB} = \frac{1}{2}$$

or

$$ad = 0.5 \times 2.3 = 1.15 \text{ cm}$$

Draw *os* || *DE*, *de* ⊥ *DE*.

$$v_{sd} = ds = 1.8 \text{ cm} = 0.18 \text{ m/s}$$
$$v_s = gs = 4.4 \text{ cm} = 0.44 \text{ m/s}$$
$$\frac{de}{ds} = \frac{DE}{DS} = 150/45$$

or

$$de = 1.8 \times \frac{150}{45} = 6 \text{ cm}$$
$$v_{ed} = 6 \text{ cm} = 0.6 \text{ m/s}$$
$$\omega_{DE} = \frac{v_{de}}{DE} = \frac{0.6}{0.1} = 6 \text{ rad/s (ccw)}$$

Locate *e*. Draw *ef* ⊥ *EF* and *if* || *XX*.

$$of = v_f$$
$$ef = v_{fe} = 6.6 \text{ cm} = 0.66 \text{ m/s}$$

Table 2.3

Link	Length r m	Velocity v m/s	Normal acceleration $f^n = v^2/r$ m/s²	Tangential acceleration $f^t = \alpha \cdot r$ m/s²	Coriolis acceleration $f^{cr} = 2v\omega$ m/s²
OA	0.025	0.523	10.94	–	–
AB	0.150	0.23	0.352	–	–
BC	0.06	0.46	2.52	–	–
DE	0.15	0.6	2.4	–	–
EF	0.10	0.66	4.356	–	–
DS	0.045	0.18	0.72	–	–
G fixed	–	0.44	–	–	5.28
S on link					

Table 2.3 shows the calculation of Coriolis acceleration.

$$f_{sg}^{cr} = 2v_{sg}\,\omega_{DE} = 2 \times 0.44 \times 6 = 5.28 \text{ m/s}^2$$

The acceleration diagram is shown in Fig.2.52(c) to a scale of 1 cm = 1 m/s^2.

$$oa \parallel OA = 10.94 \text{ m/s}^2$$
$$ab' = f_{ba}^{n} \parallel BA = 0.352 \text{ m/s}^2$$
$$b'b \perp ab' = f_{ba}^{n} \quad \text{or} \quad BA$$
$$cb'' = f_{bc}^{n} \parallel BC = 3.52 \text{ m/s}^2$$
$$b''b \perp cb'' = f_{bc}^{t}$$

Locate point b. Join ob and ab.

$$ab = f_{ba}; \quad ob = f_b$$
$$\frac{ad}{ab} = \frac{AD}{AB}$$

or

$$ad = 0.5 \times 6.5 = 3.25 \text{ cm}$$
$$ds' \parallel DS = f_{sd}^{n} = 0.72 \text{ m/s}^2$$
$$s's \perp ds'$$

Draw $gs' = f_{sg}^{cr} = 5.28$ m/s^2 $\perp SD$.

$$s''s \parallel SD$$

Locate s. Join os and sd.

The acceleration of sliding in the trunnion, $s''s = f_{sg}^{s} = 0.3$ cm = 0.3 m/s^2

2.8 KLEIN'S CONSTRUCTION

Klein's construction is used to draw the acceleration diagram for a single slider–crank mechanism. The following steps may be followed to draw the acceleration diagram, as shown in Fig.2.53:

1. Draw the configuration diagram OAB of the slider–crank mechanism.
2. Draw $OI \perp OB$ and produce BA to meet OI at C.
3. With AC as the radius and A as the centre, draw a circle.
4. Find the midpoint D of AB. With D as centre and DA as radius, draw the circle to intersect the previously drawn circle at H and E. Join HE intersecting AB at F.
5. Produce HE to meet OB at G. Join GA. Then $OAFG$ is the acceleration diagram.

$$OA = f_{ao}^{n}$$
$$AF = f_{ba}^{n}$$

The acceleration of piston (slider B),

$$f_b = \omega^2 \cdot OG$$
$$f_{ba}^{t} = \omega^2 \cdot FG$$
$$f_{ba}^{n} = \omega^2 \cdot AF$$
$$f_{ba} = \omega^2 \cdot AG$$

6. To find the acceleration of any point X in AB, draw a line $XX \parallel OB$ to intersect GA at X. Join OX. Then,

$$f_x = \omega^2 \cdot OX$$
$$\alpha_{AB} = \frac{f_{ba}^{t}}{AB} = \omega^2 \cdot \frac{FG}{AB} \text{ (ccw)}$$

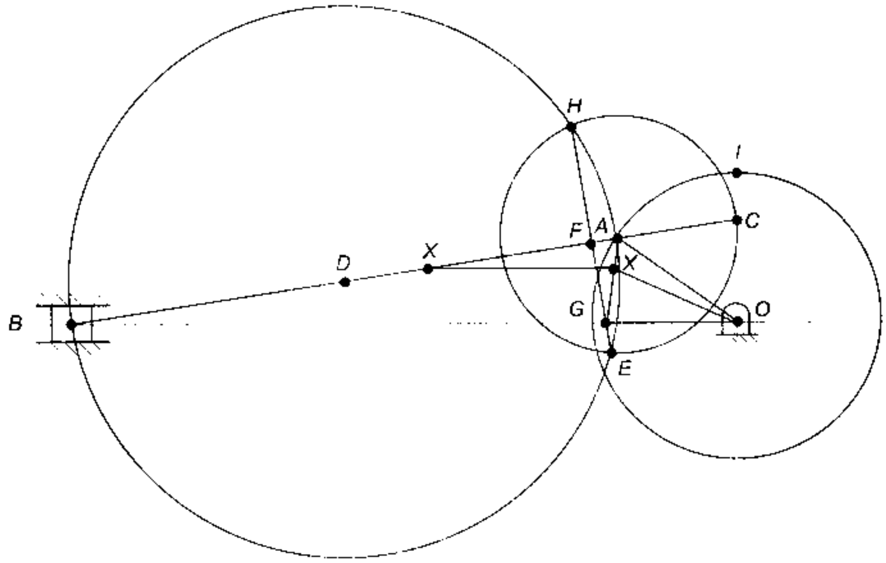


Fig.2.53 Klein's construction

Example 2.26

The crank of an engine 250 mm long rotates at a uniform speed of 240 rpm. The ratio of connecting rod length to crank radius is 4. Determine (a) the acceleration of the piston, (b) the angular acceleration of the rod, and (c) the acceleration of a point *X* on the connecting rod at 1/3rd length from crank pin. The crank position is 30° from inner dead centre.

■ Solution

The acceleration diagram using Klein's construction has been drawn in Fig.2.54.

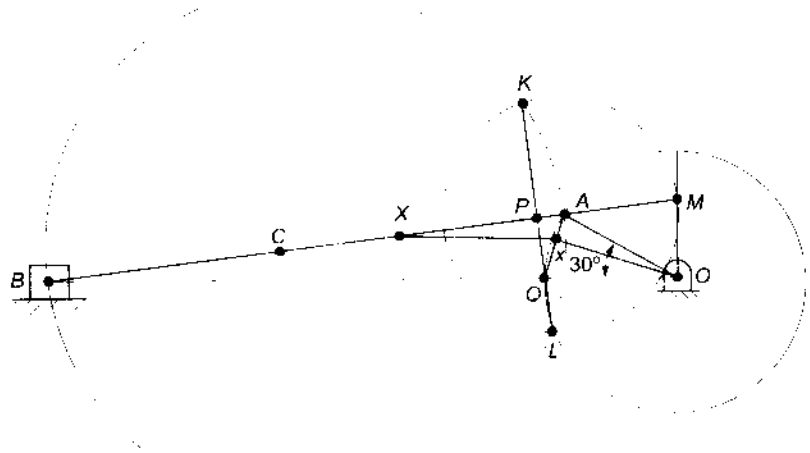


Fig.2.54 Klein's construction

Draw $OM \perp BO$. The acceleration diagram is shown by OAPQ.

$$AP = f_{ba}^{n}$$
$$PQ = f_{ba}^{t}$$
$$AQ = f_{ba}$$

$$\omega = 2 \times \frac{240}{60} = 25.13 \text{ rad/s}$$

Draw Xx parallel to BO. Join AQ and Ox. Then,

$$Ox \propto f_x$$
$$f_b = \omega^2 \cdot OQ$$

(a) Acceleration of the piston $\omega^2 \times OQ = (25.13)^2 \times 2.5 \times 10 = 15788 \text{ cm/s}^2$

(b) Tangential acceleration of the rod $= \omega^2 \times PQ = (25.13)^2 \times 1.25 \times 10 = 7894 \text{ cm/s}^2$

Angular acceleration of the rod $= \frac{7894}{100} = 78.94 \text{ rad/s}^2$

(c) Acceleration of the point X on the rod $= \omega^2 \times Ox = (25.13)^2 \times 2.4 \times 10 = 15156 \text{ cm/s}^2$

2.9 ANALYTICAL ANALYSIS OF SLIDER–CRANK MECHANISM

Consider the slider–crank mechanism shown in Fig.2.55. Let θ be the angle turned through by the crank $OA = r$ when the slider B has moved by an amount x to the right, and ϕ the angle which the connecting rod $AB = l$ makes with the line of stroke.

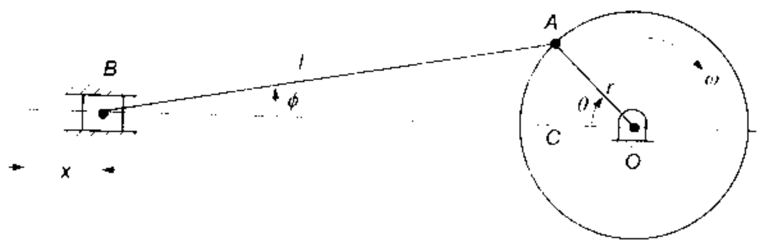


Fig.2.55 Slider–crank mechanism

$$x = r + l - (OC + BC)$$
$$= r + l - (r\cos\theta + l\cos\phi)$$
$$= r(1 - \cos\theta) + l(1 - \cos\phi)$$

Now,
$$AC = r\sin\theta = l\sin\phi$$
$$\sin\phi = \left(\frac{r}{l}\right)\sin\theta$$

Let,
$$n = \frac{l}{r}$$

Then,
$$\cos\phi = \left[1 - \sin^2\phi\right]^{0.5}$$
$$= \left[1 - \frac{\sin^2\theta}{n^2}\right]^{0.5}$$

$$
\begin{aligned}
x &= r\left[(1-\cos\theta)+l\left\{1-\left(1-\frac{\sin^2\theta}{n^2}\right)^{0.5}\right\}\right]\\
&= r\left[(1-\cos\theta)+n\left\{1-\left(1-\frac{\sin^2\theta}{n^2}\right)^{0.5}\right\}\right]\\
&= r\left[(1-\cos\theta)+n\left\{1-\left(1-\frac{\sin^2\theta}{2n^2}-\cdots\right)\right\}\right]\\
&\approx r\left[(1-\cos\theta)+\frac{\sin^2\theta}{2n}\right]\\
&\approx r\left[(1-\cos\theta)+\frac{(1-\cos 2\theta)}{4n}\right] \qquad (2.27)
\end{aligned}
$$

Velocity of the slider,

$$
\begin{aligned}
v_b &= \frac{dx}{dt}=\frac{dx}{d\theta}\cdot\frac{d\theta}{dt}\\
&= \frac{dx}{d\theta}\cdot\omega\\
&= \omega r\left[\sin\theta+\frac{\sin 2\theta}{2n}\right] \qquad (2.28)
\end{aligned}
$$

Acceleration of the slider,

$$
\begin{aligned}
f_b &= \frac{d^2x}{dt^2}\\
&= \frac{dv_b}{dt}\\
&= \frac{dv_b}{d\theta}\cdot\frac{d\theta}{dt}\\
&= \omega\cdot\frac{dv_b}{d\theta}\\
&= \omega^2 r\left[\cos\theta+\frac{\cos 2\theta}{n}\right]
\end{aligned}
$$

Now,

$$
\begin{aligned}
\sin\phi &= \frac{\sin\theta}{n}\\
\cos\phi\cdot\frac{d\phi}{dt} &= \cos\theta\cdot\frac{d\theta}{dt}\cdot\frac{1}{n}\\
&= \omega\frac{\cos\theta}{n}
\end{aligned}
$$

Angular velocity of the connecting rod,

$$
\begin{aligned}
\omega_{ba} &= \frac{d\phi}{dt}=\left(\frac{\omega}{n}\right)\cdot\left(\frac{\cos\theta}{\cos\phi}\right)\\
&= \frac{\omega\cos\theta}{\left(n^2-\sin^2\theta\right)^{0.5}}\\
&\approx \left(\frac{\omega}{n}\right)\cos\theta \qquad (2.29)
\end{aligned}
$$

Angular acceleration of the connecting rod.

$$\alpha_{ba} = \frac{d\omega_{ba}}{dt} = -\omega^2 \sin\theta \frac{(n^2-1)}{(n^2-\sin^2\theta)^{\frac{3}{2}}}$$

$$\approx -\left(\frac{\omega^2}{n}\right)\cdot \sin\theta \qquad (2.30)$$

Example 2.27

In Example 2.26, calculate analytically the acceleration of the piston and the angular acceleration of the rod.

■ **Solution**

$$n = \frac{l}{r} = 4, \theta = 30°$$

Acceleration of the piston, $f_b = \omega^2 r\left[\cos\theta + \frac{(\cos 2\theta)}{n}\right] = (25.13)^2 \times 25\left[\cos 30° + \frac{(\cos 60°)}{4}\right]$

$$= 15646 \text{ cm/s}^2$$

Angular acceleration of the rod $\cong \left(\frac{\omega^2}{n}\right)\sin\theta = [(25.13)^2 \times \sin 30°]/4 = 78.94 \text{ rad/s}^2$

2.10 COMPLEX MECHANISMS

With the inclusion of ternary or higher order floating link to a simple mechanism, the successive application of the relative velocity and relative acceleration equations fail to complete the analysis. Such a mechanism is classified as kinematically complex mechanism.

Low degree of complexity When a complex mechanism can be rendered simple by a change of input link, it is called a mechanism having low degree of complexity. In the mechanism shown in Fig.2.56, when the input link is 2, then the velocity and acceleration of point *B* cannot be determined from the velocity and acceleration of point *A*, as the radius of path of curvature of point *B* is unknown. However, with the input link as link 6 or link 5, the velocity and acceleration of *C* and *D* can be determined . Then by the image of ternary link *BCD*, the velocity and acceleration of *B* is determined.

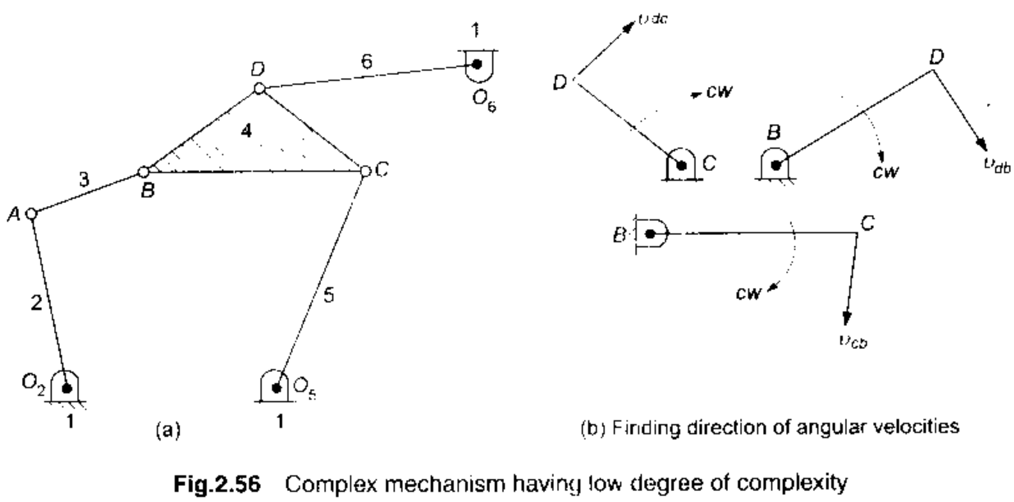
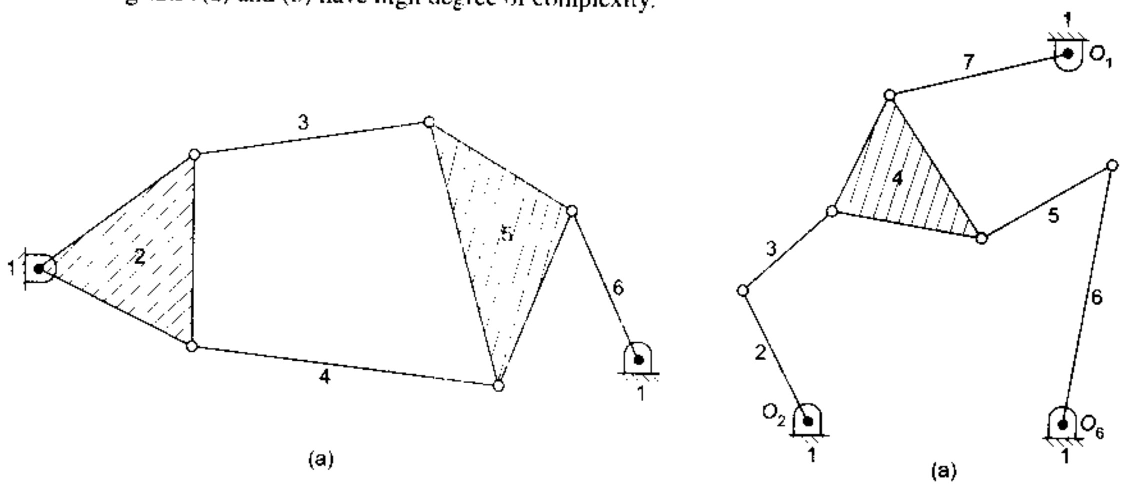


Fig.2.56 Complex mechanism having low degree of complexity

High degree of complexity When a complex mechanism cannot be rendered simple by a change of input link, it is called a mechanism with high degree of complexity. In such a mechanism the radii of the paths of curvature of two or more motion transfer points of a floating link are not known. The mechanisms shown in Figs.2.57(a) and (b) have high degree of complexity.

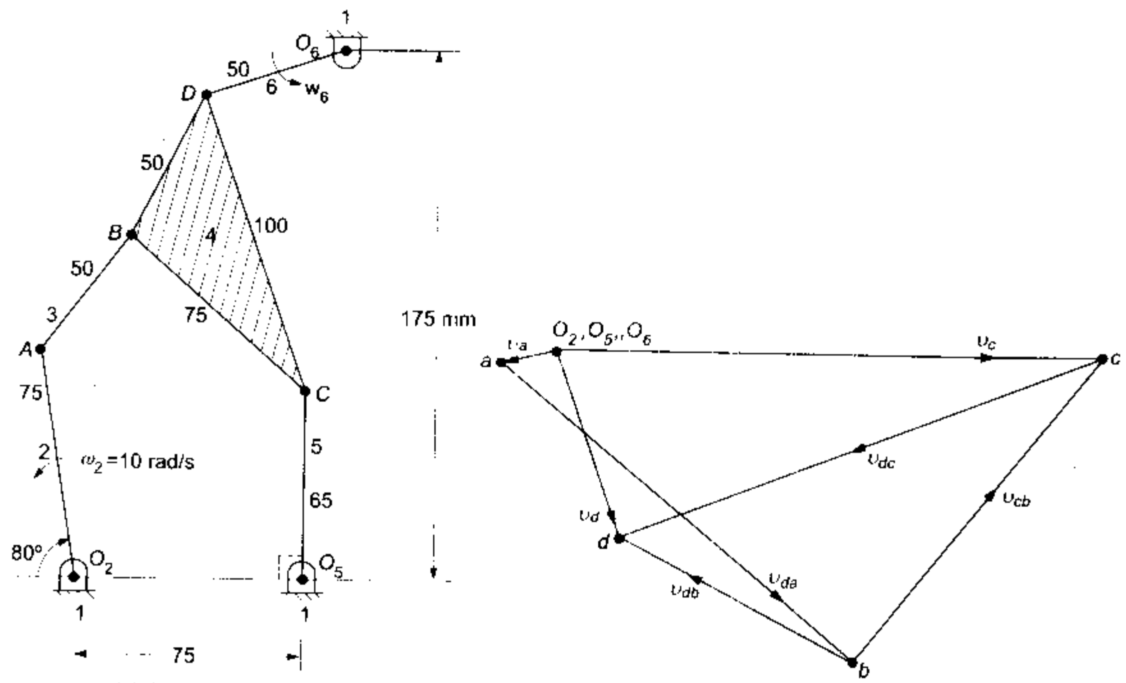


(a) (a)

Fig.2.57 High degree of complexity mechanisms

Example 2.28

For the mechanism shown in Fig.2.58(a), determine ω_4 and ω_6.



(a) Configuration diagram (b) Velocity diagram

Fig.2.58

■ Solution

With link 2 as the input link, the mechanism is complex. Make link 6 the input link. Then it becomes a simple mechanism.

Let $\omega_6 \cdot O_6D = 25$ mm $= v_d$.

(a) Draw $o_6d \perp O_6D = v_d = 25$ mm, as shown in Fig.2.58(b).

Locate point c. $o_5c \perp O_5C$, $cd \perp CD$

(b) Draw $cb \perp BC$, $db \perp BD$

Locate point b.

(c) Draw $o_2a \perp O_2A$, $ab \perp AB$

Locate point a.

(d) Measure $o_2a = v_a$. The direction of the velocity vector v_a confirms the direction of rotation of crank O_2A in the counter-clockwise direction. If it does not confirm, the solution is repeated with link 6 rotating in the clockwise direction.

By measurement, $o_2a = 8$ mm.

$$\omega_2 \cdot O_2A = 8 \times \text{scale}$$

or

$$\text{scale, 1 mm} = 10 \times \frac{75}{8} = 92.75 \text{ mm/s}$$

$$\omega_6 = \frac{o_6d \times \text{scale}}{O_6D} = \frac{25 \times 92.75}{50} = 46.875 \text{ rad/s (ccw)}$$

$$\frac{v_{dc}}{DC} = \frac{v_{db}}{DB} = \frac{v_{cb}}{BC}$$

or

$$\frac{cd}{DC} = \frac{bd}{BD} = \frac{bc}{BC}; \quad \omega_4 = \frac{70 \times 92.75}{100} = 65.625 \text{ rad/s (cw)}$$

Example 2.29

For the mechanism shown in Fig.2.59, determine ω_6.

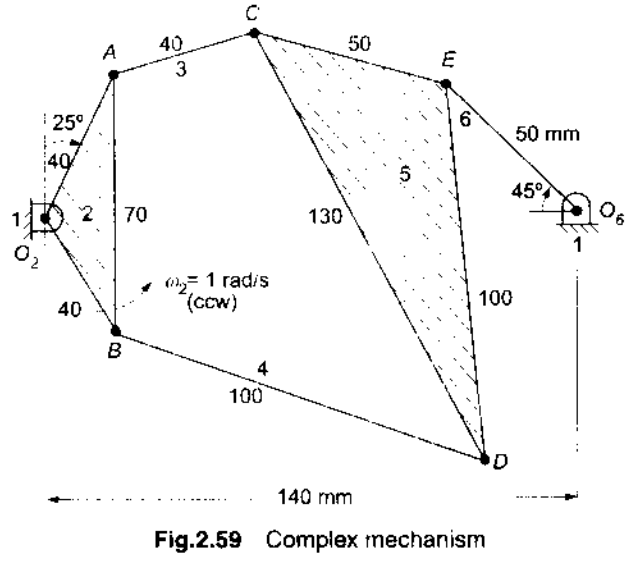


Fig.2.59 Complex mechanism

Solution

With link 1 being the fixed link and link 2 the input link, the velocity diagram cannot be drawn. It is a complex mechanism with a of high degree of complexity.

To draw the velocity diagram, draw the mechanism with link 4 as the fixed link, as shown in Fig.2.60(a). Now the velocity diagram can be drawn for *BACD* and points O_2 and *E* can be located on the velocity image of BAO_2 and *DCE*, as shown in Fig.2.60(b). After locating points O_2 and *E*, point O_6 can be located.

$$v_a = \omega_{24} \cdot AB$$

Let $ba = 50$ mm, then v_a = Diagram scale × 50. The velocity diagram is drawn as follows:

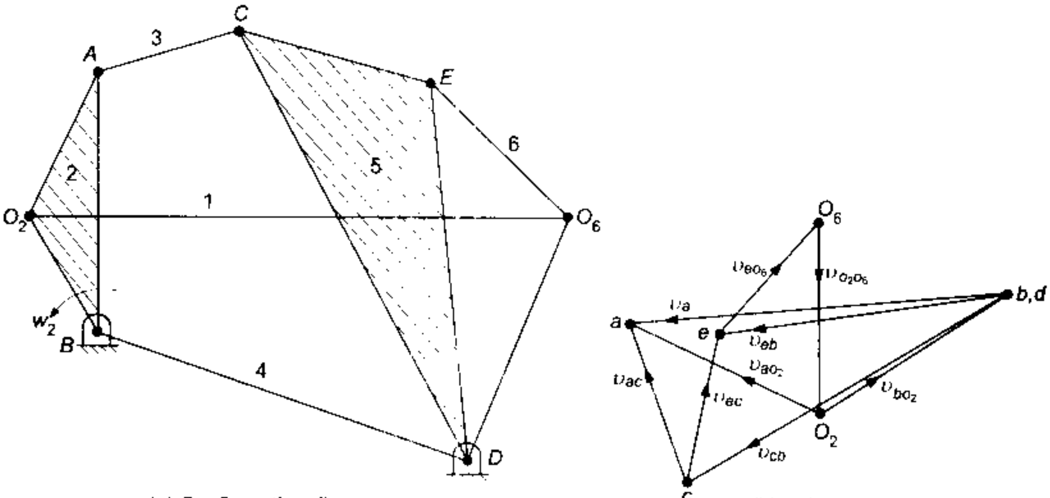
(a) Draw $ab \perp AB = 50$ mm.

(b) Draw $ac \perp AC$ and $dc \perp CD$ to locate *c*.

(c) Draw $ce \perp CE$ and $de \perp DE$ to locate *e*.

(d) Draw $ao_2 \perp AO_2$ and $bo_2 \perp BO_2$ to locate o_2.

(e) Draw $eo_6 \perp EO_6$ and $o_2o_6 \perp O_2O_6$ to locate o_6.



(a) Configuration diagram (b) Velocity diagram

Fig.2.60

$$\omega_{24} = \text{Diagram scale} \times \frac{ba}{AB}$$

$$= \text{Scale} \times \frac{50}{70} = 0.7143 \times \text{Scale (ccw)}$$

$$v_{o6o2} = O_2O_6 \times \omega_{14} = 140 \times \omega_{14}$$

$$= o_2o_6 \times \text{scale}$$

or

$$\omega_{14} = \frac{25 \times \text{scale}}{140}$$

$$= 0.1786 \times \text{Scale rad/s}$$

$$v_{o6e} = O_6E \times \omega_{64} = 50 \times \omega_{64}$$

$$= eo_6 \times \text{Scale}$$

or $$\omega_{64} = \frac{eo_6 \times \text{Scale}}{50} = \frac{20 \times \text{Scale}}{50}$$
$$= 0.4 \times \text{Scale rad/s (ccw)}$$

Similarly, $$\omega_{34} = \frac{ac \times \text{Scale}}{\text{AC}} = \frac{22 \times \text{Scale}}{40}$$
$$= 0.55 \times \text{Scale rad/s (ccw)}$$
$$\omega_{54} = \frac{dc \times \text{Scale}}{\text{DC}} = \frac{49 \times \text{Scale}}{130}$$
$$= 0.377 \times \text{scale rad/s (ccw)}$$

But, $$\omega_2 = \omega_{21} = 1 \text{ rad/s (ccw)}$$

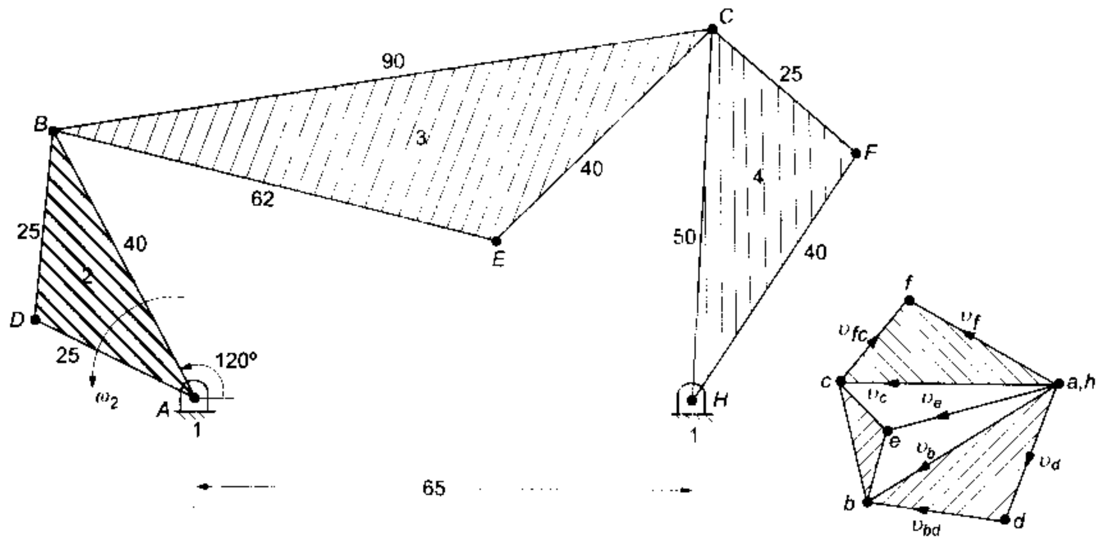
Also, $$\omega_{21} = \omega_{24} - \omega_{14}$$
$$= (0.7143 - 0.1786) \times \text{Scale rad/s}$$

or $$1 = 0.5357 \times \text{Scale rad/s}$$
$$\text{scale} = 1.867 \text{ mm/s/mm}$$

Thus, $$\omega_{24} = 0.7143 \times 1.867 = 1.3336 \text{ rad/s (ccw)}$$
$$\omega_{14} = 0.1786 \times 1.867 = 0.3334 \text{ rad/s (ccw)}$$
$$\omega_{34} = 0.55 \times 1.867 = 1.0268 \text{ rad/s (ccw)}$$
$$\omega_{64} = 0.4 \times 1.867 = 0.7468 \text{ rad/s (ccw)}$$
$$\omega_{54} = 0.377 \times 1.867 = 0.7038 \text{ rad/s (ccw)}$$
$$\omega_6 = \omega_{61} = \omega_{64} - \omega_{14}$$
$$= 0.7468 - 0.3334 = 0.4134 \text{ rad/s (ccw)}$$

Example 2.30

For the mechanism, shown in Fig.2.61(a), the linear velocity of point E is 2.3 m/s. Link 2 rotates at uniform angular velocity in the counter-clockwise direction. Determine ω_2, ω_3, and ω_4.



(a) Configuration diagram

(a) Velocity diagram

Fig.2.61

Solution

The magnitude of ω_2 is unknown. Let us take $ab = 25$ mm (arbitrarily).

Draw the velocity diagram as shown in Fig.2.61(b).

$$ab \perp AB = 30 \text{ mm}$$
$$bd \perp BD$$
$$ad \perp AD$$

Locate *d*.

$$bc \perp BC$$
$$hc \perp HC$$

Locate *c*.

$$be \perp BE$$
$$ce \perp CE$$

Locate *e*. Join *ae*.

$$cf \perp CF$$
$$hf \perp HF$$

Locate *f*.

$$v_e = \text{Scale of diagram} \times ae$$
$$2300 = \text{Scale} \times 23$$

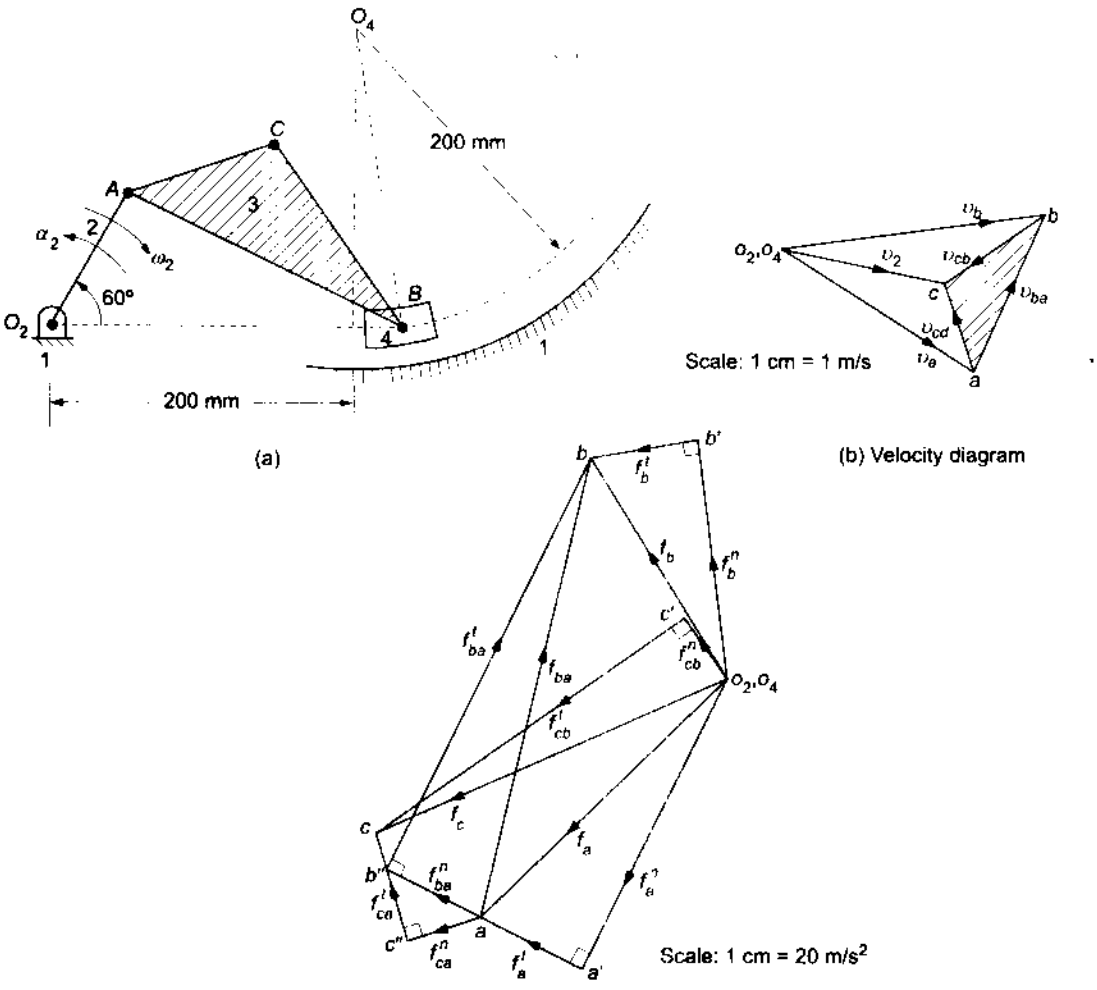
or

$$\text{scale} = \frac{100 \text{ mm/s}}{\text{mm}}$$

$$\omega_2 = \frac{v_b}{AB} = \frac{ab \times \text{scale}}{AB} = \frac{30 \times 100}{40}$$
$$= 75 \text{ rad/s (ccw)}$$
$$\omega_3 = \frac{v_{cb}}{CB} = \frac{bc \times \text{scale}}{CB} = \frac{17 \times 100}{90}$$
$$= 18.9 \text{ rad/s (cw)}$$
$$\omega_4 = \frac{v_c}{HC} = \frac{hc \times \text{scale}}{HC} = \frac{28 \times 100}{50}$$
$$= 56 \text{ rad/s (ccw)}$$

Example 2.31

In the mechanism shown in Fig.2.62(a), the link 2 rotates with angular velocity of 30 rad/s and an angular acceleration of 240 rad/s^2. Determine (a) the acceleration of points *B* and *C*, (b) the angular accelerations of link 3 and 4 and (c) the relative acceleration α_{34}. $O_2A = 100$ mm, $AB = 200$ mm, $AC = 100$ mm and $BC = 150$ mm.



(a)

(b) Velocity diagram

(c) Acceleration diagram

Fig.2.62

■ Solution

$$v_a = 30 \times 0.1 = 3 \text{ m/s}$$

The velocity diagram is shown in Fig.2.62(b) to a scale of 1 cm = 1 m/s.

$$v_b = v_a + v_{ba}$$

$$v_c = v_a + v_{ca} = v_b + v_{cb}$$

$$v_a = o_2a \text{ and } \perp O_2A$$

$$v_b = o_2b \text{ and } \perp o_4b = 2.6 \text{ cm} = 2.6 \text{ m/s}$$

$$v_{ba} = ab \text{ and } \perp AB = 2.3 \text{ cm} = 2.3 \text{ m/s}$$

$$v_{cb} = bc \text{ and } \perp BC = 1.7 \text{ cm} = 1.7 \text{ m/s}$$

$$v_{ca} = o_2b \text{ and } \perp o_4b = 1.2 \text{ cm} = 1.2 \text{ m/s}$$

$$v_c = o_2c$$

The acceleration diagram is shown in Fig.2.62(c) to a scale of 1 cm = 20 m/s^2.

$$f_b^n = \frac{v_b^2}{O_4B} = \frac{2.6^2}{0.2} = 64.8 \text{ m/s}^2 \quad \text{(from } B \text{ to } O_4\text{)}$$

$$f_b^t \perp f_b^n$$

$$f_a^n = \frac{v_a^2}{O_2A} = \frac{3^2}{0.1} = 90 \text{ m/s}^2 \quad \text{(from } A \text{ to } O_2\text{)}$$

$$f_{ba}^n = \frac{v_{ba}^2}{AB} = \frac{2.3^2}{0.2} = 26.45 \text{ m/s}^2 \quad \text{(from } B \text{ to } A\text{)}$$

$$f_{ba}^t \perp f_{ba}^n$$

$$f_b = f_a + f_{ba}$$

$$f_b^n + f_b^t = f_a^n + f_a^t - f_{ba}^n + f_{ba}^t$$

$$f_b = o_2b = 2.5 \text{ cm} = 70 \text{ m/s}^2$$

$$f_b^t = b'b = 1.4 \text{ cm} = 28 \text{ m/s}^2$$

$$f_{ba}^t = b''b = 6.2 \text{ cm} = 124 \text{ m/s}^2$$

$$\alpha_3 = \frac{f_{ba}^t}{BA} = \frac{124}{0.2} = 620 \text{ rad/s}^2 \text{ (ccw)}$$

$$\alpha_4 = \frac{f_b^t}{O_4B} = \frac{28}{0.2} = 140 \text{ rad/s}^2 \text{ (cw)}$$

$$\alpha_{34} = \alpha_3 - \alpha_4 = 620 + 140 = 760 \text{ rad/s}^2 \text{ (ccw)}$$

The acceleration image is shown in Fig.2.62(c).

$$f_c = f_a + f_{ca}^n + f_{ca}^t$$

$$= f_b + f_{cb}^n + f_{cb}^t$$

$$f_{ca}^n = \frac{v_{ca}^2}{CA} = \frac{1.2^2}{0.1} = 14.4 \text{ m/s}^2$$

$$f_{ca}^t \perp f_{ca}^n$$

$$f_{cb}^n = \frac{v_{cb}^2}{CB} = \frac{1.7^2}{0.150} = 19.27 \text{m/s}^2$$

$$f_{cb}^t \perp f_{cb}^n$$

$$f_c = o_2c = 5 \text{ cm} = 100 \text{ m/s}^2$$

Exercises

1 The dimensions of various links for the mechanism shown in Fig.2.63 are $OA = 0.5$ m, $AB = 1.5$ m and $AC = CD = 0.9$ m. The crank OA has uniform angular speed of 180 rpm. Determine the velocities of slides B and D. Also calculate the turning moment at C if a force of 500 N acts on B and a force of 800 N acts on D, as shown.

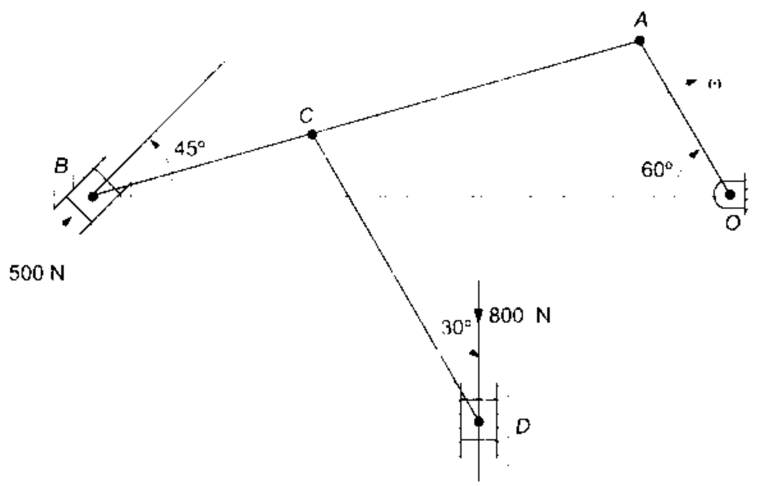


Fig.2.63 Mechanism with crank and sliders

2 The dimensions of the various links of the mechanism shown in Fig.2.64 are $OA = 50$ mm, $AB = 400$ mm, $BC = 150$ mm, $CD = 100$ mm and $DE = 250$ mm. The crank OA rotates at 60 rpm. Find the velocity of slider E.

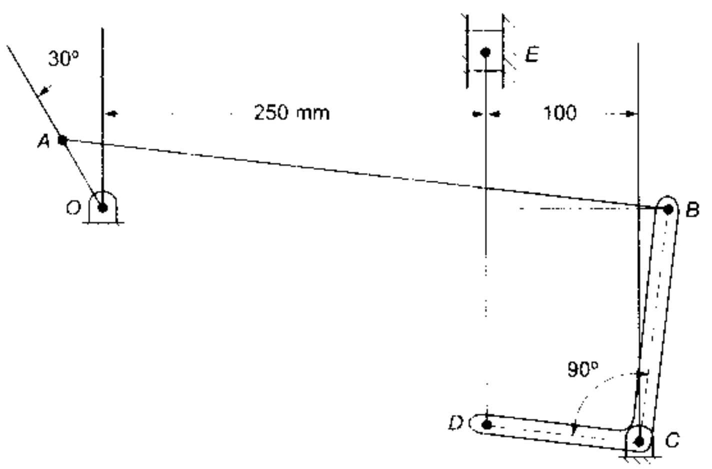


Fig.2.64 Mechanism with crank and slider

3 The dimensions of the various links of the mechanism shown in Fig.2.65 are $OA = 30$ mm, $AB = 75$ mm, $BC = 45$ mm and $BD = 100$ mm. The crank OA rotates at 120 rpm. Determine the velocity of slider D and angular speeds of links AB, BC and BD.

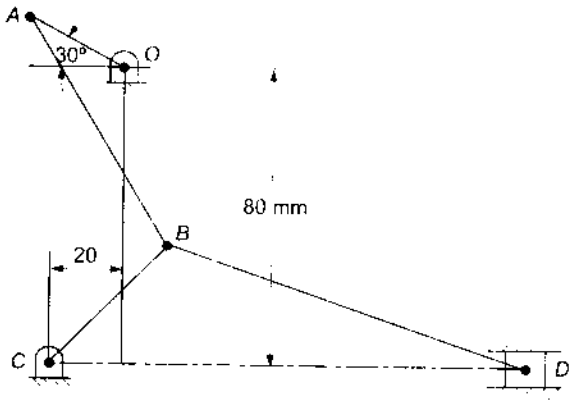


Fig.2.65 Mechanism with crank and slider

4 The crank OA of the mechanism shown in Fig.2.66 rotates at 120 rpm. The dimensions of the various links are $OA = 100$ mm, $AB = CE = 400$ mm, $AC = 125$ mm, and $EF = 300$ mm.

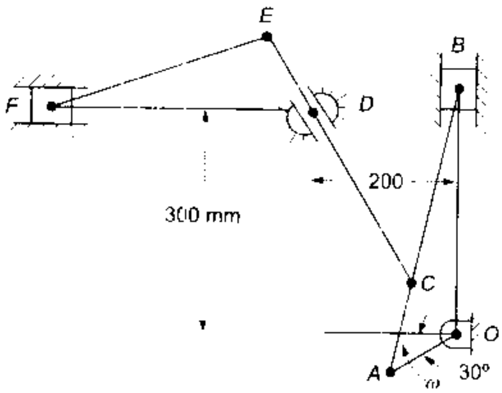


Fig.2.66 Mechanism with rod sliding in a slot in trunnion

The rod CE slides in a slot in trunnion at D. Determine (a) the velocity of F, (b) the velocity of sliding of CE in D and (c) the angular velocity of CE.

5 The crank and slotted lever mechanism is shown in Fig.2.67. The dimensions of the various links are $OA = 200$ mm, $AB = 100$ mm, $OC = 400$ mm and $CD = 300$ mm.
The crank AB rotates at 75 rpm. Determine (a) the velocity of the ram and (b) the angular speed of the slotted lever OC.

6 For the mechanism shown in Fig.2.68, determine the velocities of points C, E and F and the angular velocities of the links BC, CDE and EF. $\omega = 120$ rpm. $AB = 60$ mm, $BC = 120$ mm, $AD = 50$ mm, $CD = 100$ mm, $DE = 120$ mm, $CE = 50$ mm and $EF = 150$ mm.

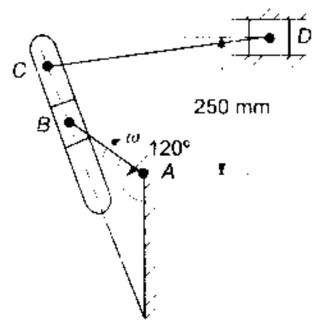


Fig.2.67 Crank and slotted lever mechanism

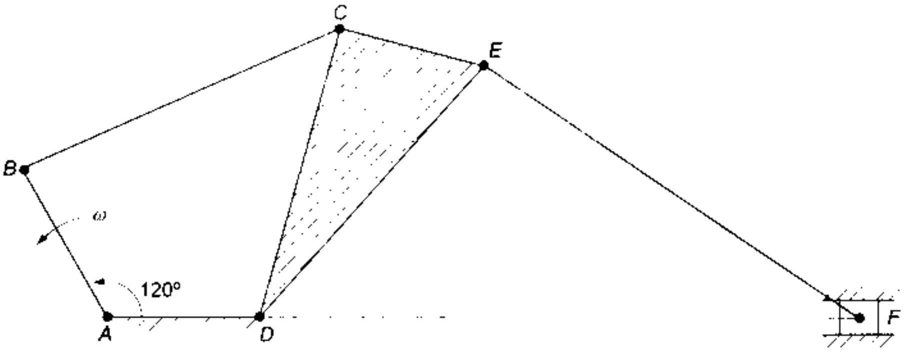


Fig.2.68 Complex mechanism

7 For the mechanism shown in Fig.2.69, determine the angular velocities of links 3 and 4 when link 2 is rotating at 120 rpm. Also find v_c and v_d.

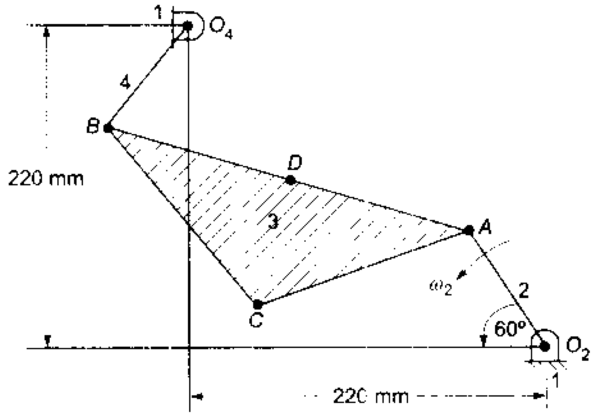


Fig.2.69 Mechanism with rotating link

O_2A = 100 mm, AB = 250 mm, AC = 150 mm, BC = 150 mm, O_4B = 100 mm and AD = 100 mm.

8 For the mechanism shown in Fig.2.70, determine the velocities of points C and A and ω_3 and ω_4. The link 2 rotates at 150 rpm. $O_2A = 380$ mm. $O_4B = 250$ mm. $AC = 250$ mm, $BC = 400$ mm and $O_2O_4 = 750$ mm.

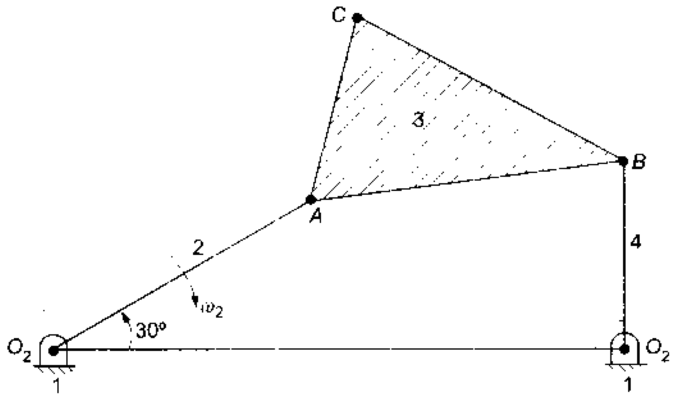


Fig.2.70 Mechanism with rotating link

9 For the mechanism shown in Fig.2.71, determine the velocity of the slider C. The link 2 rotates at 180 rpm. $O_2A = 50$ mm, $AB = 100$ mm, $AC = 200$ mm, $BD = 100$ mm, $O_6D = 100$ mm and $O_2O_6 = 100$ mm.

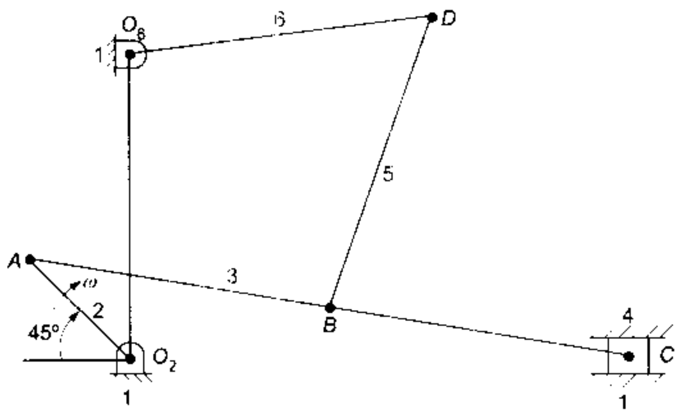


Fig.2.71 Mechanism with rotating link

10 For the crank–shaper mechanism shown in Fig.2.72, link 2 rotates at a constant angular speed of 1 rad/s. Determine the angular speed of link 4, v_{A4}, v_{A2A3} and v_{A3A4}. $O_2A = 50$ mm, $O_4A = 80$ mm and $O_2O_4 = 120$ mm.

11 For the mechanism shown in Fig.2.73, link 2 rotates at 160 rad/s. Determine v_b, ω_4 and v_{ba}. $O_2A = 150$ mm, $AB = 200$ mm and $O_4B = 150$ mm.

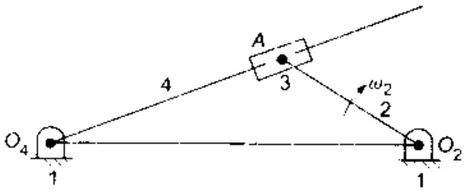


Fig.2.72 Crank–shaper mechanism

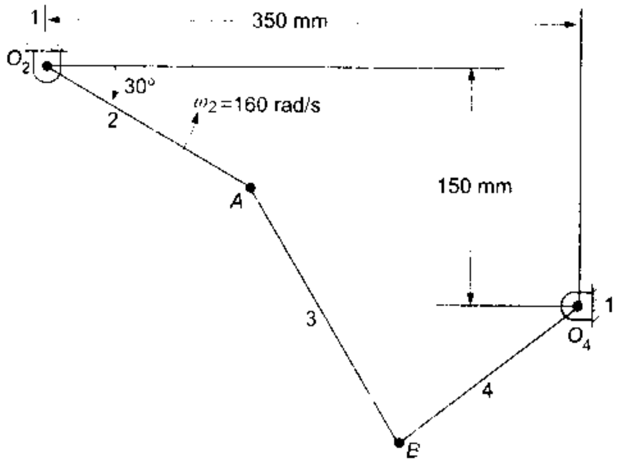


Fig.2.73 Mechanism with rotating link

12 The driving link 2 of the Whitworth quick-return mechanism shown in Fig.2.74 rotates at a constant speed of 6 m/s. Determine the velocity of the tool holder.

$O_2A = 100$ mm, $O_4B = 100$ mm, $BC = 350$ mm and $O_2O_4 = 90$ mm.

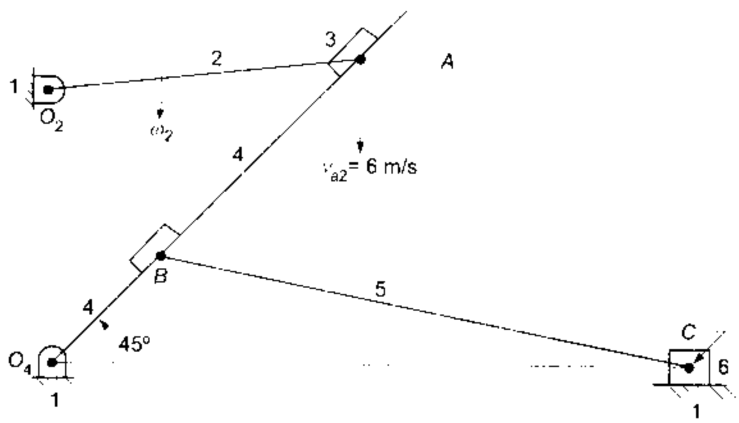


Fig.2.74 Whitworth quick-return mechanism